



# **Tensile Membrane Action of Composite Slabs in Fire**

Are the current methods really OK?

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Max beam temperature  $\sim 1150^{\circ}\text{C}$   
cf. Code critical temperature  $\sim 680^{\circ}\text{C}$





# The basis of all current simplified methods: Hayes (1968)

## Allowing for membrane action in the plastic analysis of rectangular reinforced concrete slabs

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**SYNOPSIS**

The existing methods for allowing for membrane action in predicting the ultimate behaviour of reinforced concrete slabs are reviewed and their deficiencies considered. A new equilibrium method is presented and compared with the existing methods. Comparison of the theoretical predictions of the method with its results obtained from tests on model reinforced concrete slabs shows good agreement.

**Notation**

- A, D parameters in yield line theory
- A<sub>1</sub> area of span (L<sub>1</sub>L<sub>2</sub>)
- A<sub>2</sub> area of reinforcement per unit width of slab (average value)
- A<sub>3</sub> A<sub>4</sub> parameters in yield criterion
- A<sub>5</sub> effective fibre perimeter
- A<sub>6</sub> total reinforcement equivalent stress
- A<sub>7</sub> effective depth of reinforcement
- A<sub>8</sub> reinforcement ratio
- A<sub>9</sub> reinforcement ratio in direction of span
- A<sub>10</sub> reinforcement ratio in direction of span
- A<sub>11</sub> reinforcement ratio in direction of span
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- A<sub>48</sub> reinforcement ratio in direction of span
- A<sub>49</sub> reinforcement ratio in direction of span
- A<sub>50</sub> reinforcement ratio in direction of span

**Introduction**

The behaviour of reinforced slabs in predicting the ultimate load of reinforced concrete slabs under various loading conditions has been extensively investigated. The state of the art of reinforced concrete slabs is discussed in detail in the literature, and it is clear that the existing methods for predicting the ultimate load of reinforced concrete slabs are based on the assumption that the slabs behave as rigid bodies. This assumption is not valid for slabs which are subjected to large deflections, and it is necessary to allow for the effects of membrane action in the plastic analysis of such slabs.

Magazine of Concrete Research - Vol. 28, No. 11, November 1968

**Figure 1. Reinforcement grid on a slab.**

Methods for allowing for the effect of finite membrane action on ultimate loads have been proposed by Taylor<sup>1</sup>, Kemp<sup>2</sup> and Moxham<sup>3</sup>. These methods are presented and their limitations outlined before the development of a new equilibrium method.

**Existing methods of analysis**

**TAYLOR'S METHOD**

First observations of the behaviour of one-way slabs, Taylor suggested that the two-dimensional analysis of simply supported square slabs could be extended by allowing for the increase in the effective depth of the reinforcement brought about by a redistribution of the concrete reinforcement. From this suggestion, the present author developed a simple procedure for computing slabs with sufficient and suitable reinforcement, including the possible extension to the case of the slab due to its maximum depth of the reinforcement. The author pointed out the strength of the slab after a certain finite reinforcement has been used.

**KEMP'S METHOD**

Kemp proposed a more rigorous approach which takes into account the effects of the reinforcement on the lateral area along the yield lines. The procedure of the lateral area along the yield lines was determined by a redistribution of generalised considerations and in plate equilibrium. The position of the neutral axis having been determined, the yield criterion was used to estimate moments and maximum fibres along the yield lines, and the yield load was found from equilibrium. This method allows a continuous strength reduction relationship.

**MOXHAM'S METHOD**

The first two methods of the data are only valid in regions limited to slabs due to the state of reinforcement along the yield lines. If the loading conditions of the slabs are not uniform, it is necessary to use the method of Moxham. The method of Moxham has been extended to the case of slabs (Figure 2), but is more complicated in the analysis of rectangular slabs (Figure 2).

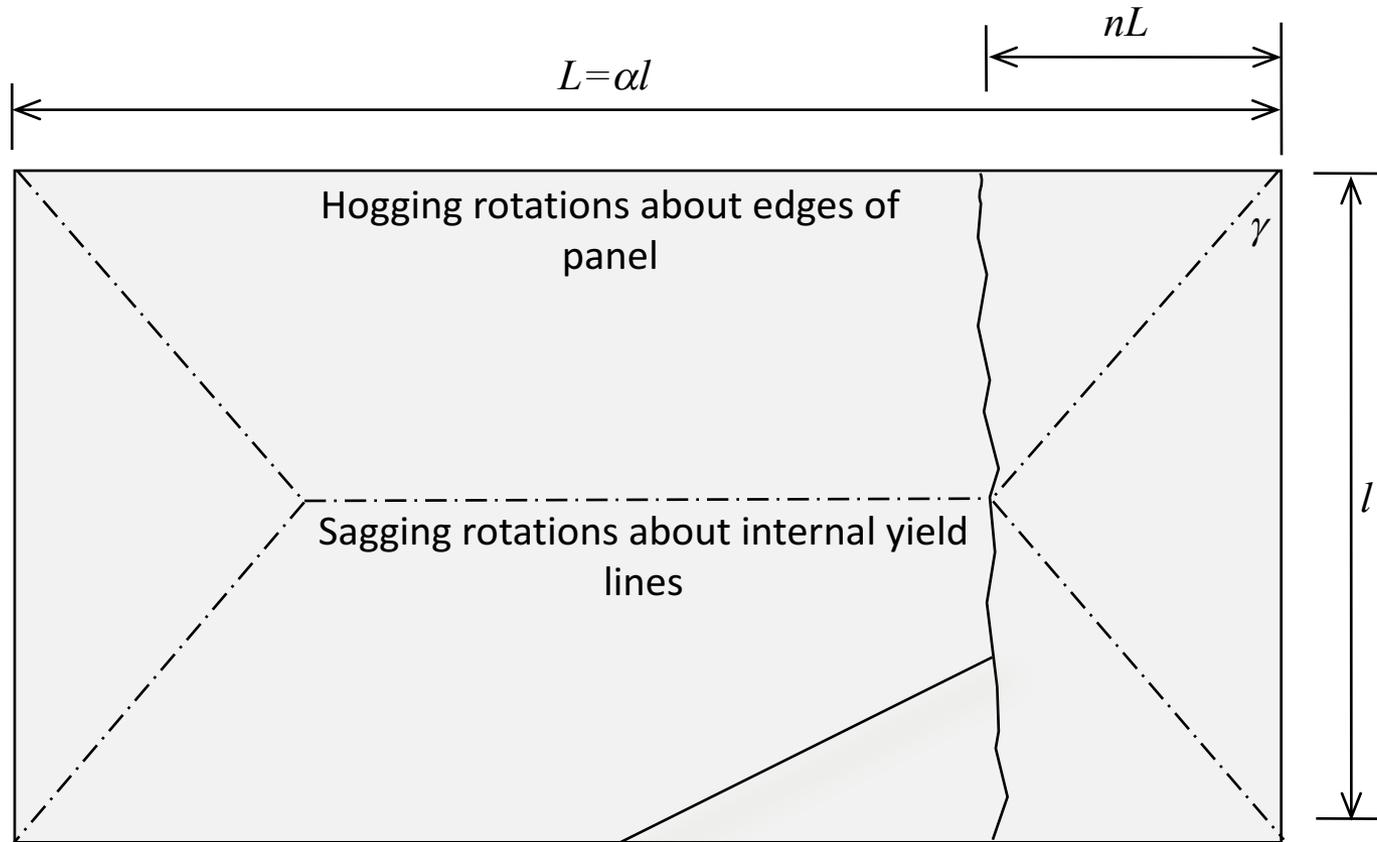
The method discussed above is not valid for the analysis of slabs which are subjected to the possibility of a yield line in the slab. For the method which generalised Taylor's method, a single approach was adopted, in which the membrane and bending effects were considered separately. Moxham's original method considered membrane reinforcement only. The two methods presented by the author to cover the case of orthotropically reinforced rectangular slabs.

Extending Taylor's original method, the reinforcement of the slab is assumed to be the yield load is given by

$$M = \frac{1}{2} (1 + \frac{1}{2} \frac{A_1}{A_2}) W \dots (1)$$



# Small-deflection yield-line mechanism – slab only

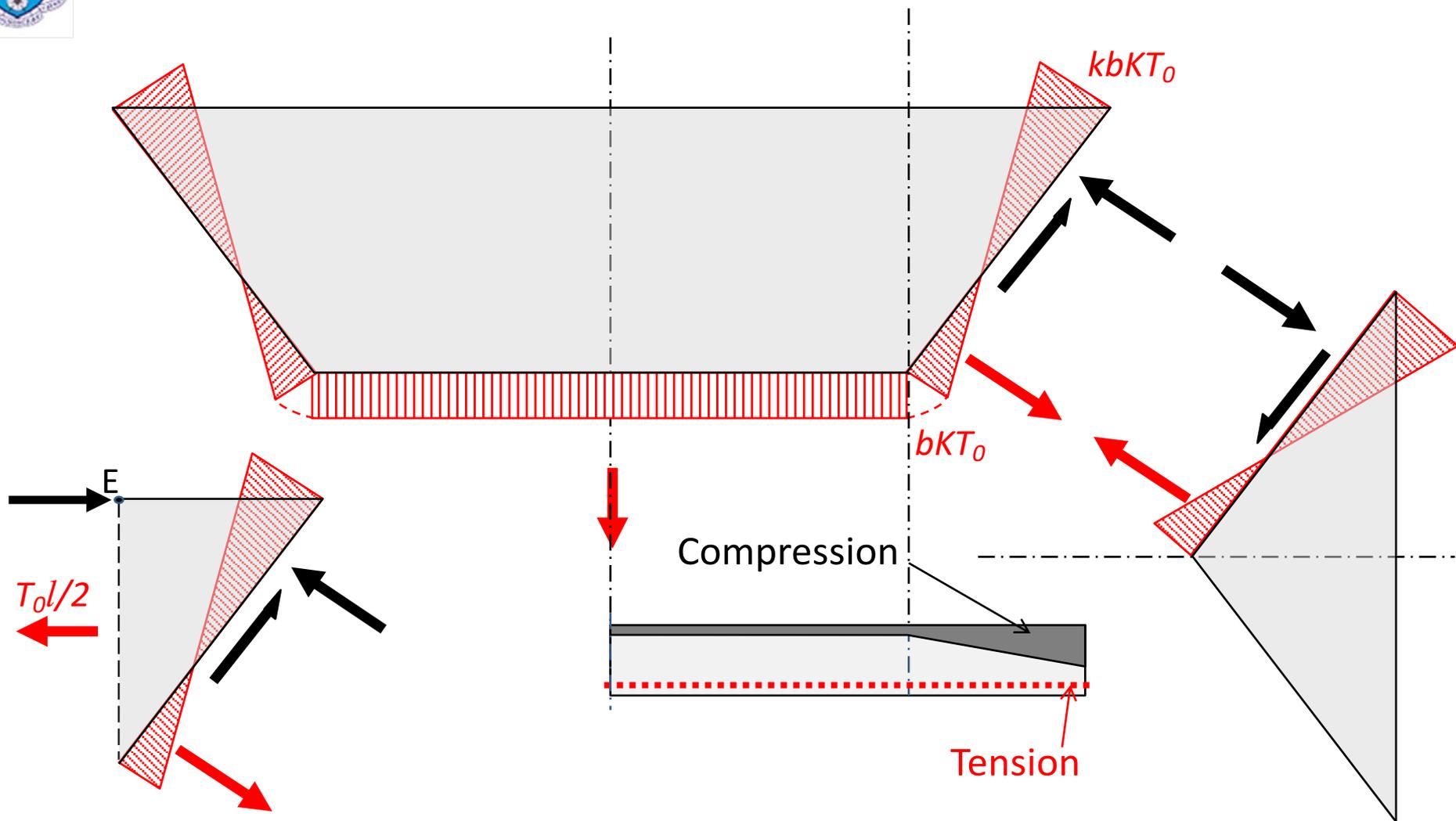


Large-deflection failure crack sometimes observed in tests and used by Hayes.

Yield-line pattern is optimized for minimum concrete slab failure load.



# Equilibrium 1 – no through-depth YL cracks - Hayes

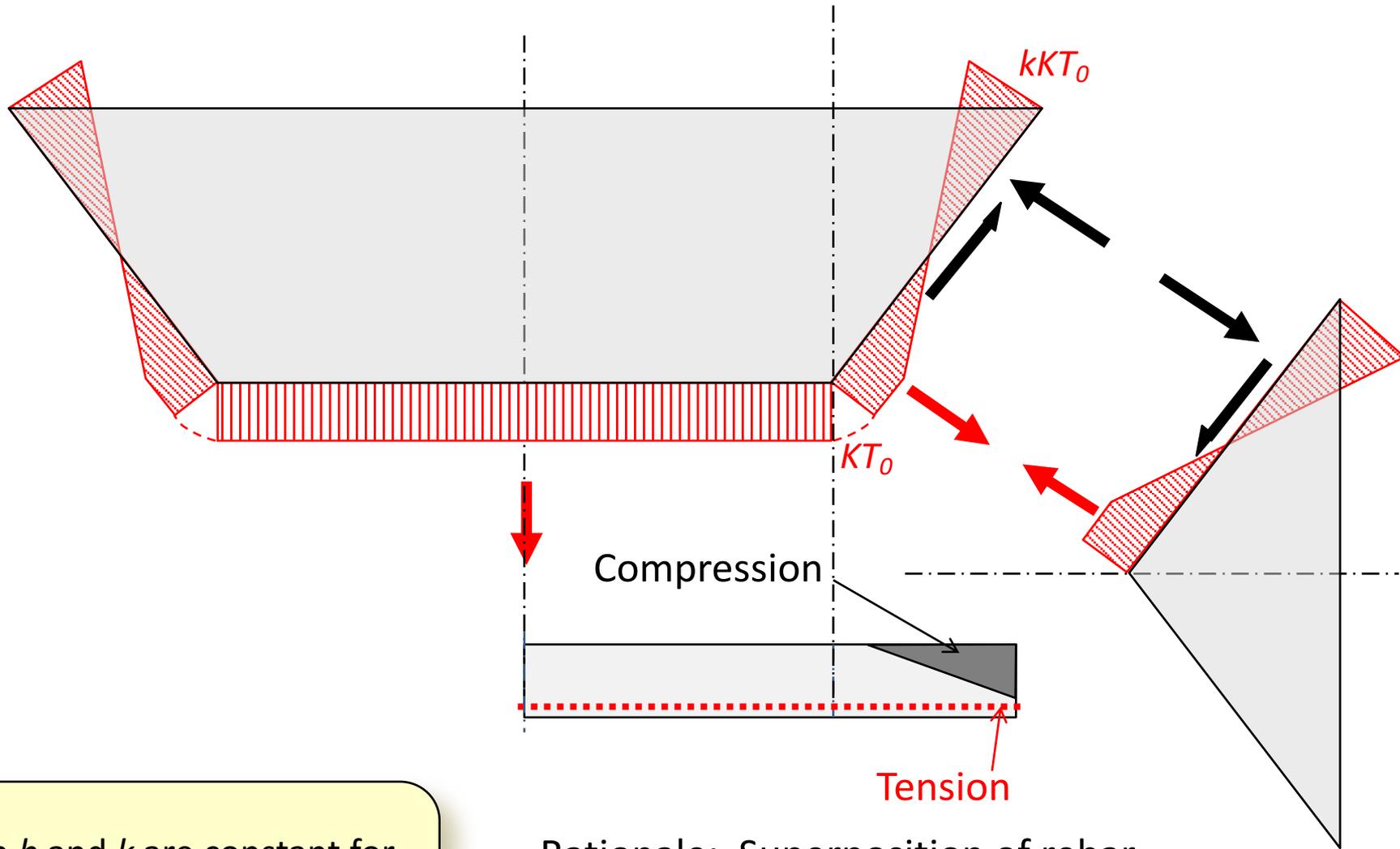


Criterion: Cracks from intersection. Moment equilibrium about E. Finds  $b$  and  $k$ .

Rationale: Superposition of rebar tension and concrete compression force/unit length.



# Equilibrium 2 – some through-depth YL cracks - Hayes

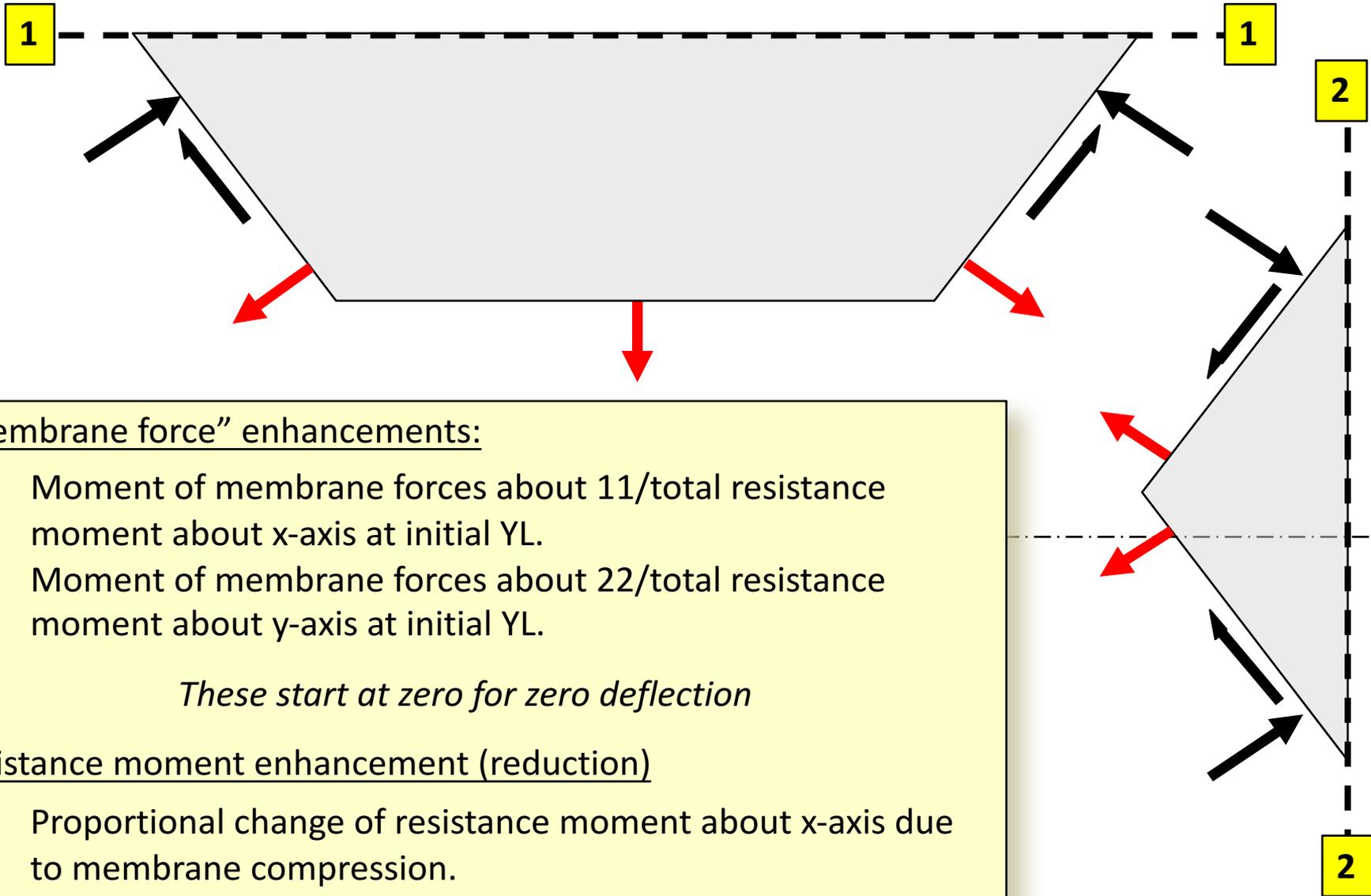


Both  $b$  and  $k$  are constant for each of the 2 cases. No variation with deflection.

Rationale: Superposition of rebar tension and concrete compression force/unit length.



# Partial enhancement factors – both cases - Hayes



## “Membrane force” enhancements:

$e_{1m}$  Moment of membrane forces about 11/total resistance moment about x-axis at initial YL.

$e_{2m}$  Moment of membrane forces about 22/total resistance moment about y-axis at initial YL.

*These start at zero for zero deflection*

## Resistance moment enhancement (reduction)

$e_{1b}$  Proportional change of resistance moment about x-axis due to membrane compression.

$e_{2b}$  Proportional change of resistance moment about y-axis due to membrane compression.



# Bending “enhancements” - Hayes

Wood’s equation for reduction of moment capacity of a rectangular RC cross-section due to axial compression:

1. Long-span reinforcement:  $\frac{M}{M_0} = 1 + A \frac{N}{T_0} + B \left(\frac{N}{T_0}\right)^2$

2. Short-span reinforcement:  $\frac{M}{M_0} = 1 + A' \frac{N}{KT_0} + B' \left(\frac{N}{KT_0}\right)^2$

These are integrated in  $x$ - and  $y$ - directions respectively for the bending moments across the yield lines for Portions 1 and 2.



# Forming an overall enhancement factor - Hayes

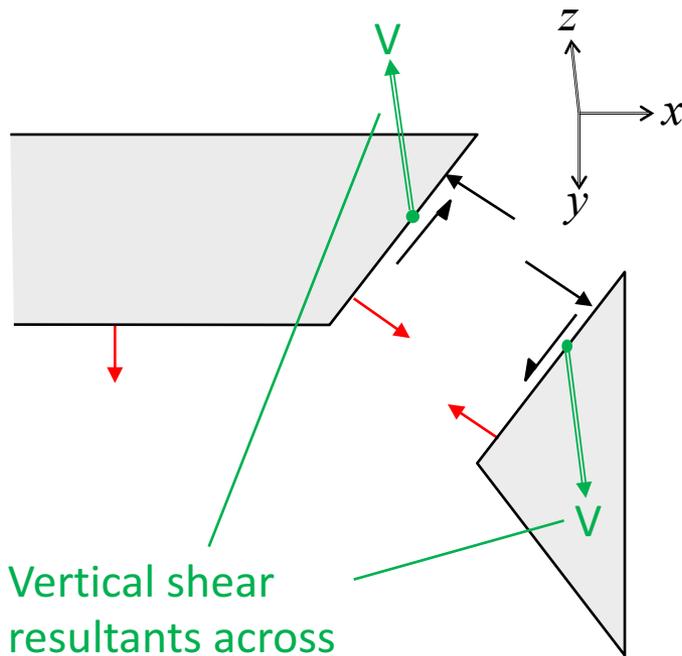
$$e_1 = e_{1m} + e_{1b}$$

$$e_2 = e_{2m} + e_{2b}$$

These are nearly always unequal (**WHY?**). Put together as

Overall enhancement factor

$$e = e_1 - \frac{e_1 - e_2}{1 + 2\mu a^2}$$



Vertical shear resultants across yield lines

These don't include any vertical shear between the facets. If these are included there is only one enhancement factor.

*(Tony Gillies 2015)*

New enhancement Factor equivalent to

$$e = e_1 - \frac{e_1 - e_2}{1 + 2\mu a^2}$$



- The membrane traction distribution is an assumption. It corresponds to unfractured mesh and either:
  - No through-depth cracks along yield lines.
  - Partial through-depth cracks along yield lines.
- Both of these distributions apply only to the case where a lateral through-depth crack has formed across the short span through the YL intersection.
- Distribution is fixed for each case. Enhancement factor starts below 1.0 – actually at zero.
- Internal forces don't depend on deflection.



# Structural fire resistance methods for composite floors



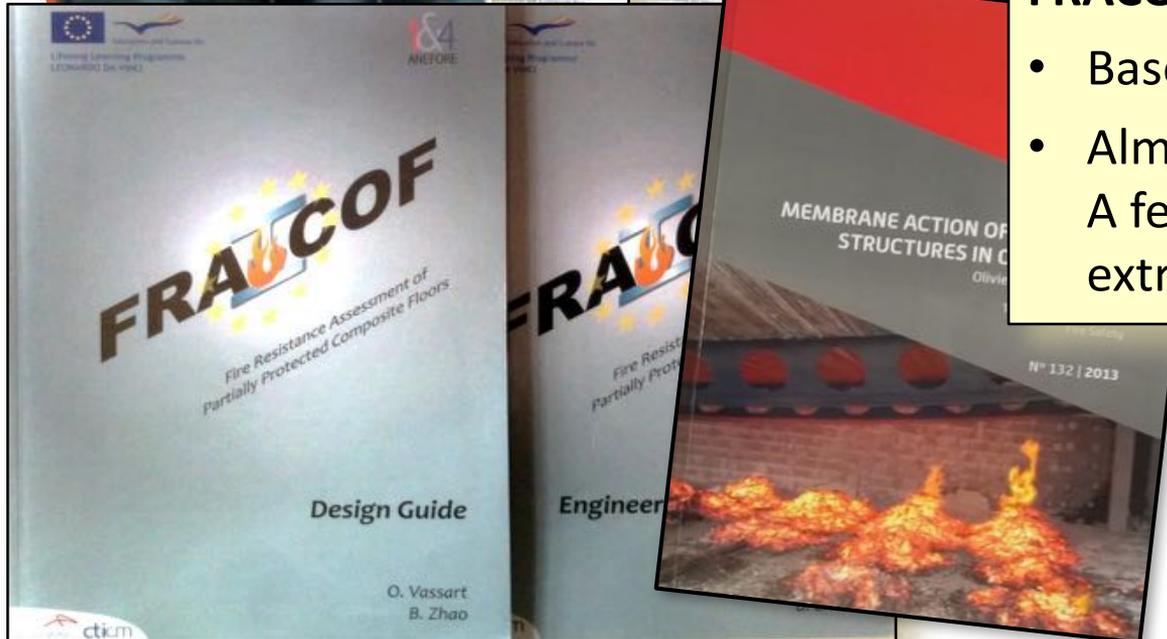
**BRE Method (Bailey 2000)**

- Amended version of Hayes's method.
- *Fire Safe Design (SCI P288)* checked using BRE-Bailey design method.

**New Zealand SPM (Clifton 2006)**

**FRACOF (2011)**

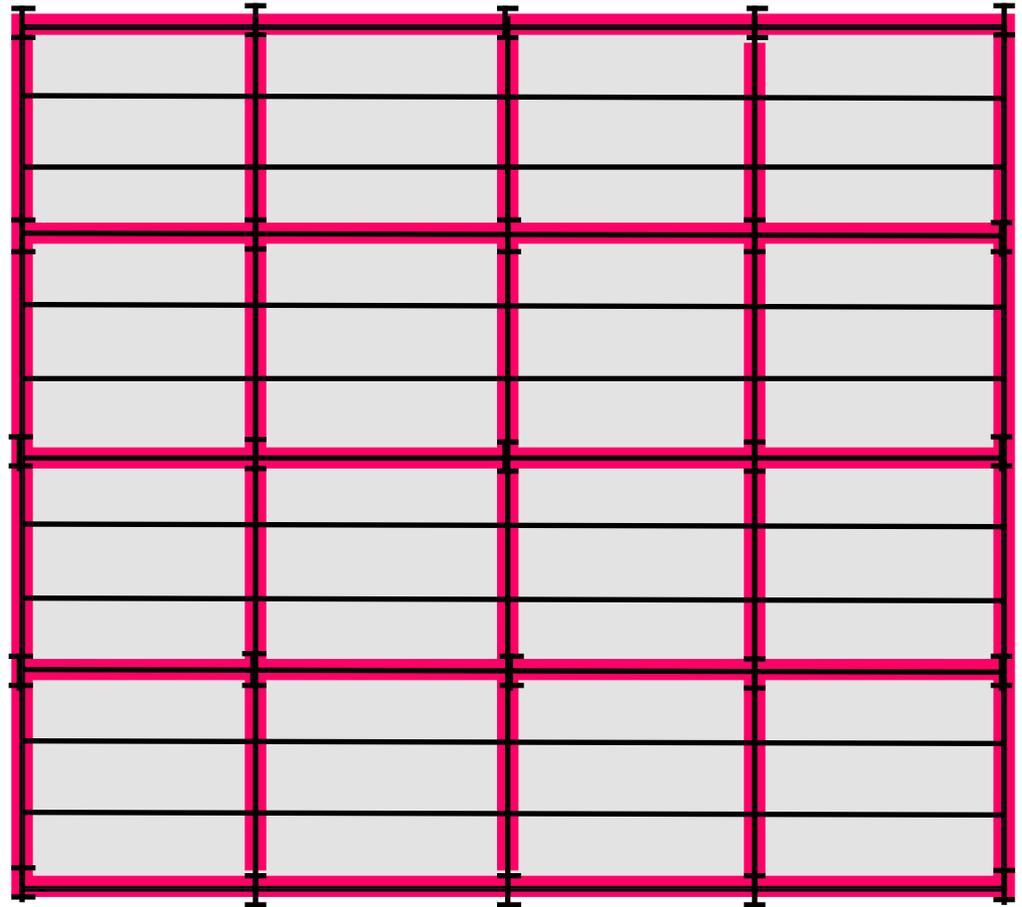
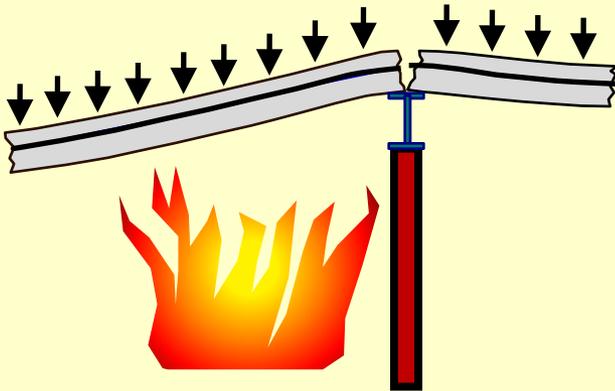
- Based on a European project.
- Almost identical to BRE method. A few changes to safety factors, extra deflection check.





# Typical design strategy for TMA

- Protect members on column gridlines.
- Leave intermediate secondary beams unprotected.
- Design individual panels without continuity.



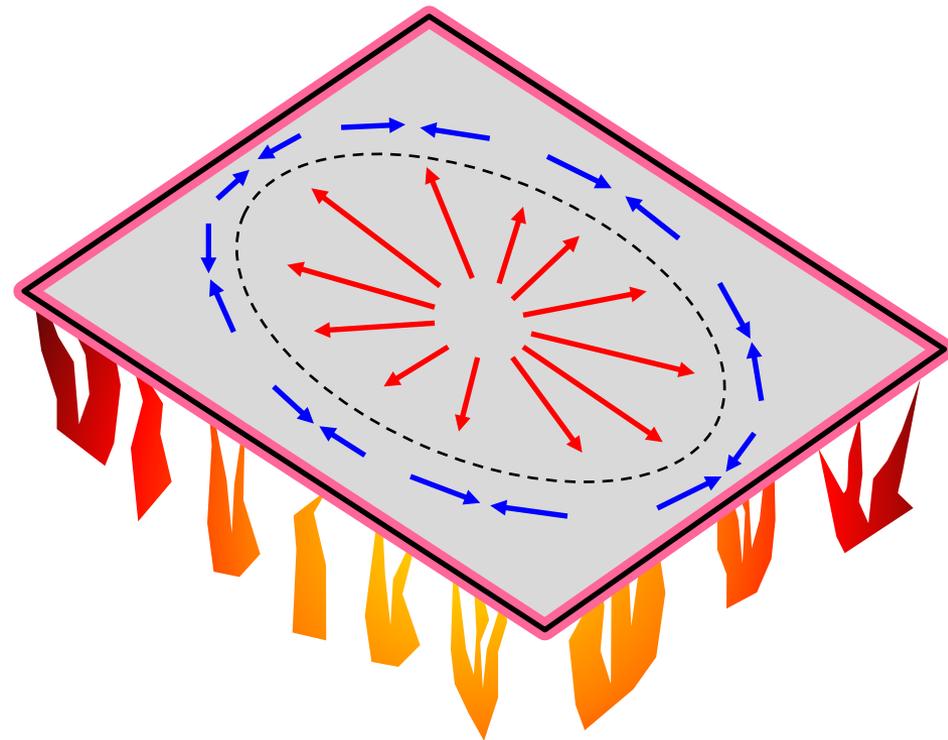
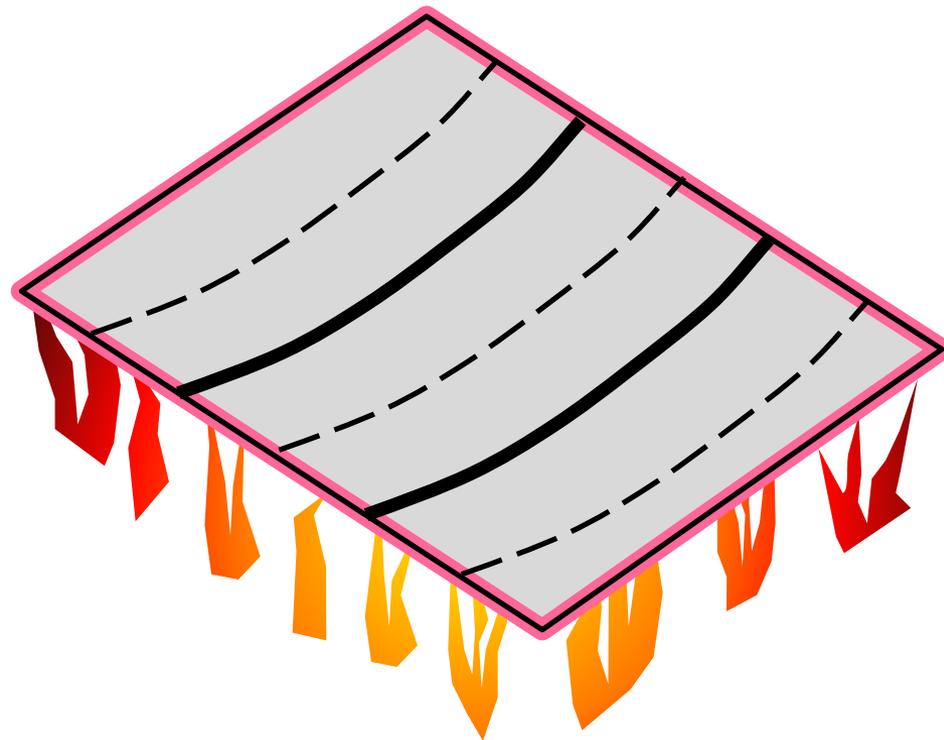


# BRE/FRACOF method

- Unprotected composite beams at high temperature carry some of the load as simply supported.

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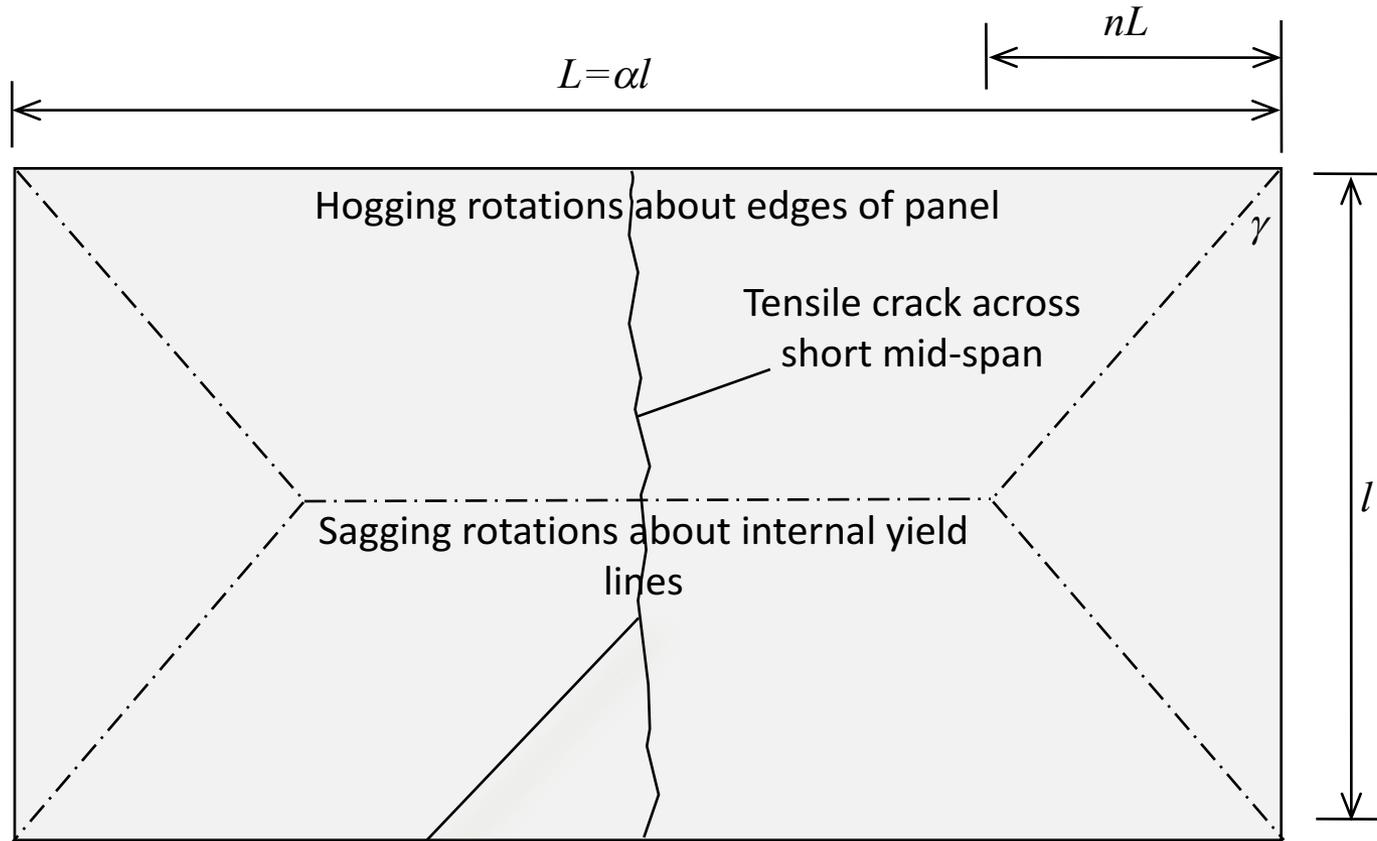
- Concrete slab carries remaining load in tensile membrane action. Needs enough deflection.





# Small-deflection yield-line mechanism – slab only

## BRE/FRACOF

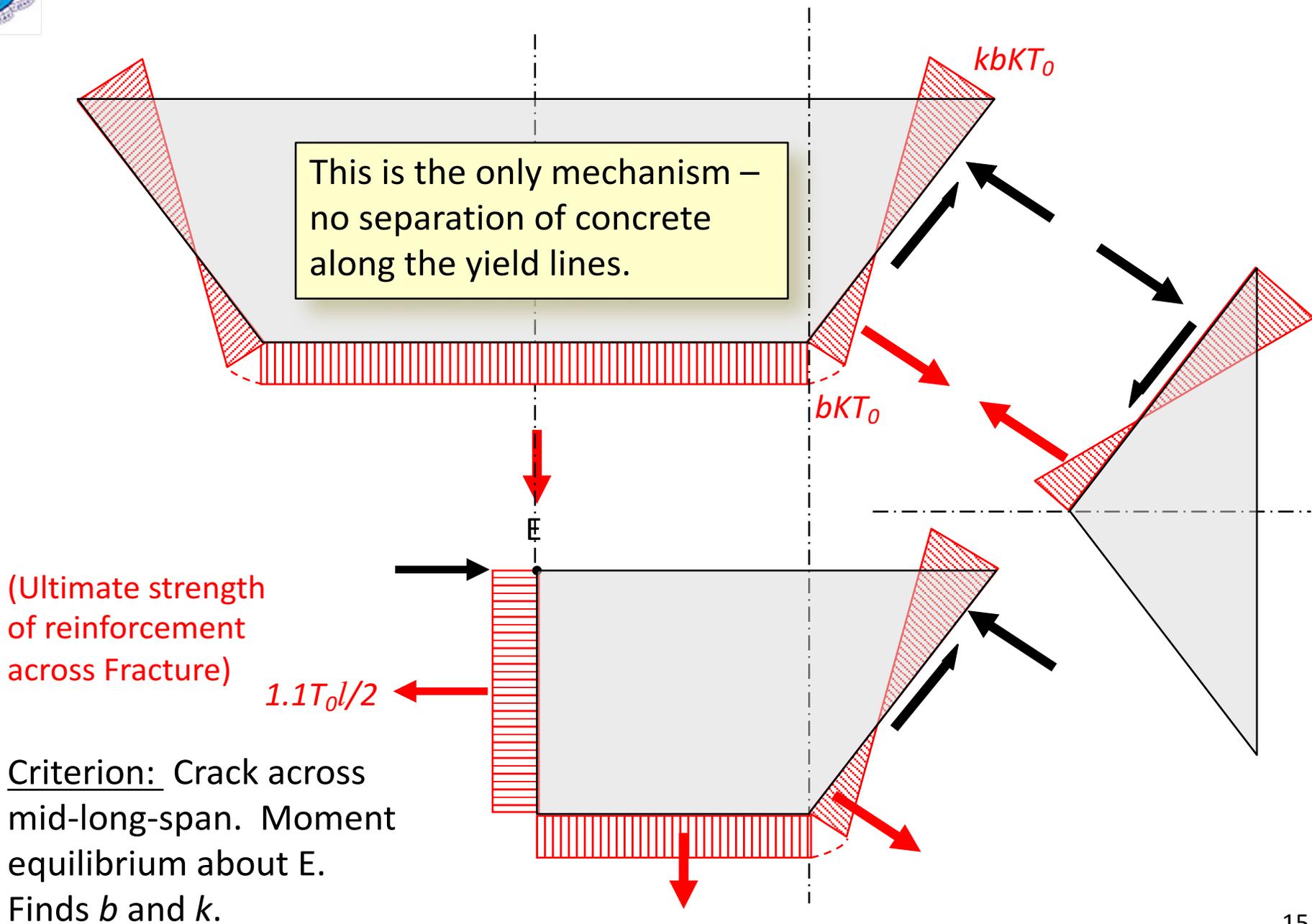


Large-deflection failure crack observed in tests and used in Bailey/BRE, FRACOF and NZ SPM.

The analysis is based on the optimal yield-line pattern for the concrete slab without considering the steel beams.



# Force equilibrium – no through-depth YL cracks – BRE etc

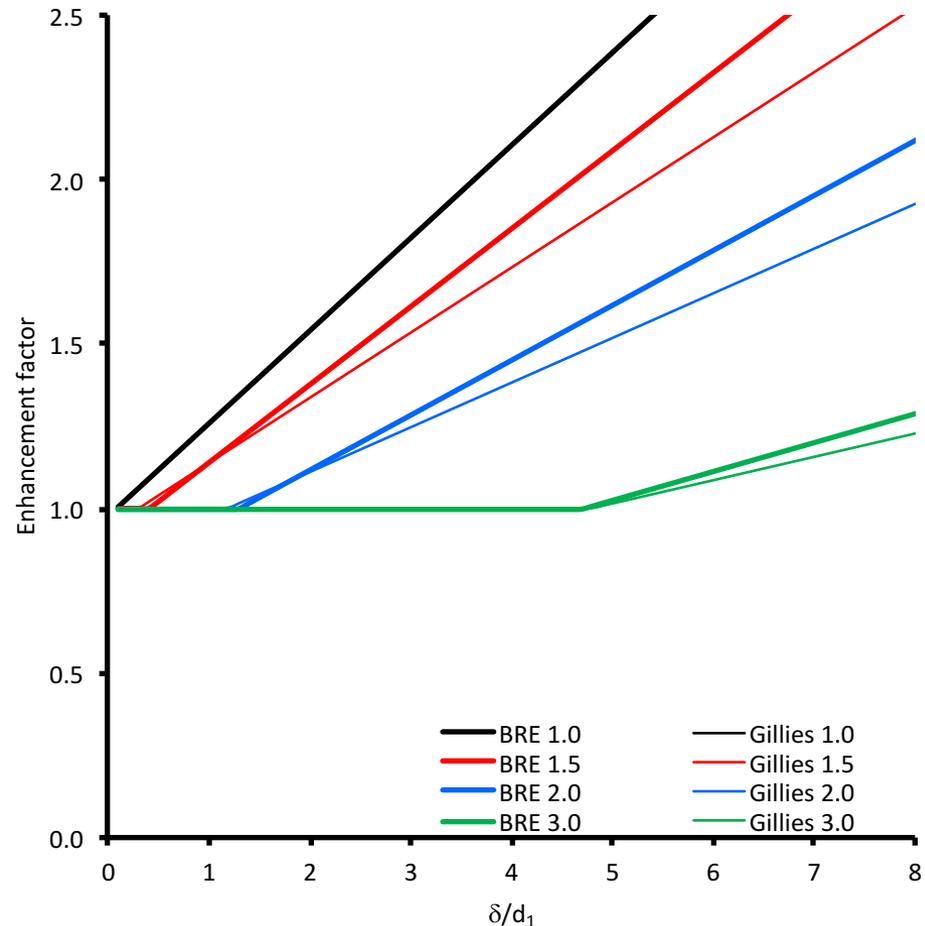




# TMA enhancement calculations – BRE/FRACOF

## Similarly to Hayes:

- Horizontal force equilibrium assuming mid-span crack. (But only the linear membrane traction distribution).
- Separate “membrane” enhancements  $e_{1m}$  and  $e_{2m}$  by moments about long and short edges.
- Add “bending” enhancements  $e_{1b}$  and  $e_{2b}$  to make  $e_1$  and  $e_2$ .
- Overall enhancement factor 
$$e = e_1 - \frac{e_1 - e_2}{1 + 2\mu\alpha^2}$$
- ... or Gillies 
$$e = e_1 - \frac{e_1 - e_2}{1 + 2\mu\alpha^2}$$
- Cutoff at enhancement 1.0 for aspect ratios > 1.0.





# Limiting deflection (central cracking) criterion

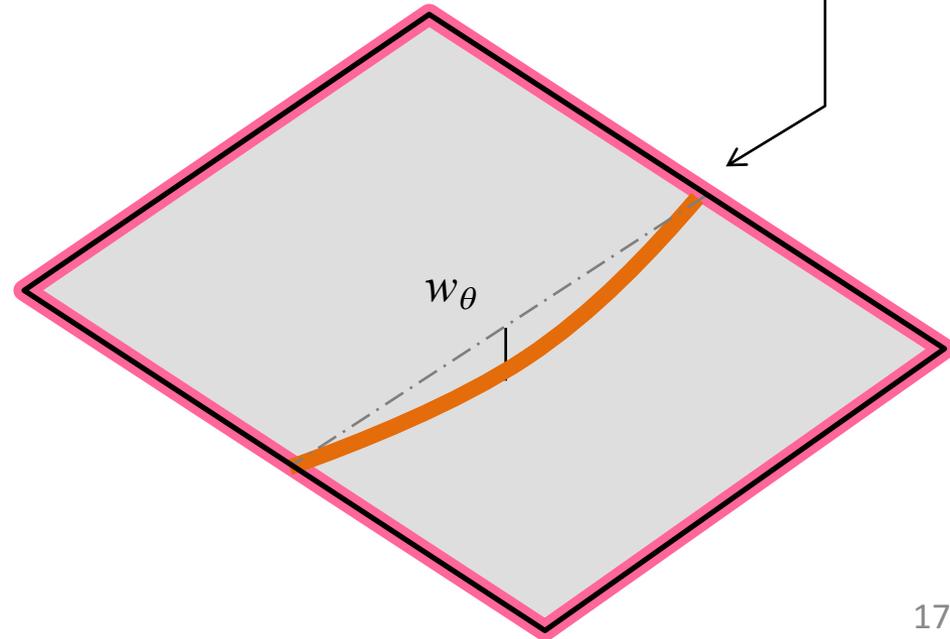
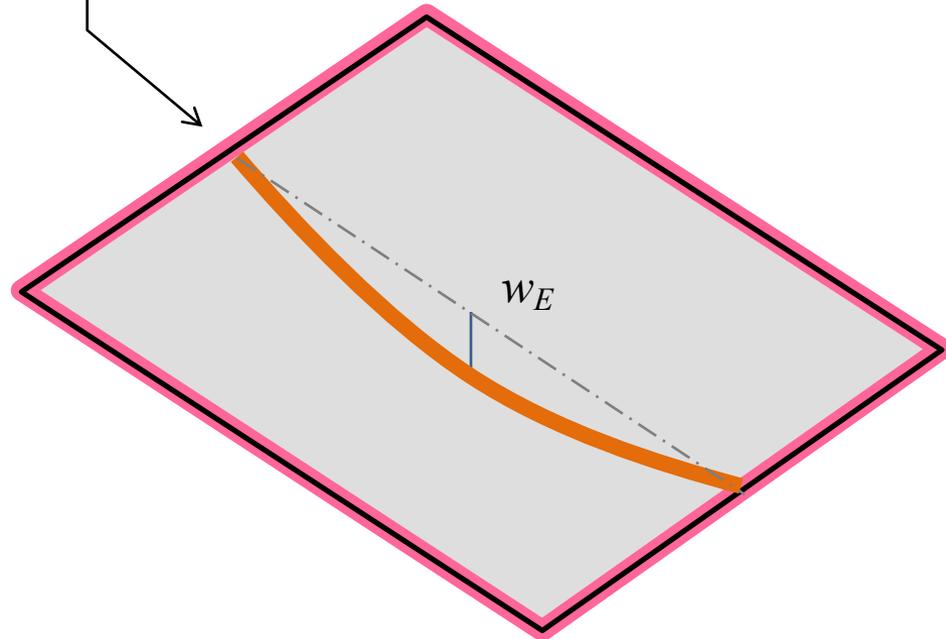


$$w_E = \sqrt{\left(\frac{0.5f_{sy}}{E_s}\right) \frac{3L^2}{8}}$$



$$w_\theta = \frac{\alpha(T_2 - T_1)l^2}{16h}$$

+





# Back to basics

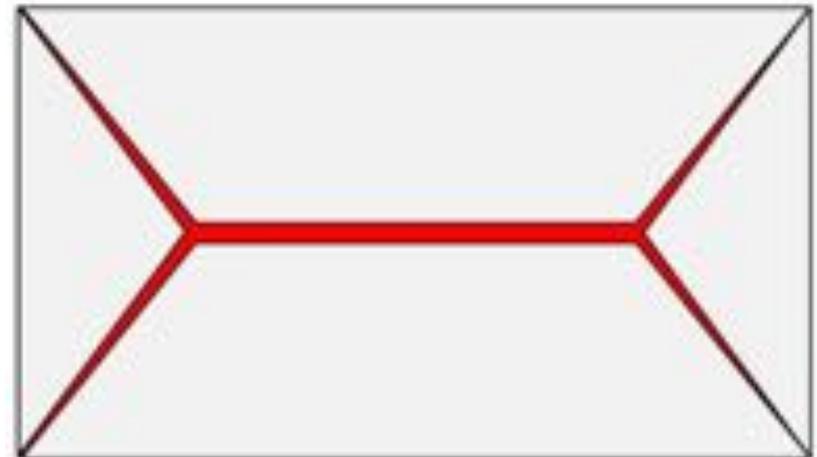
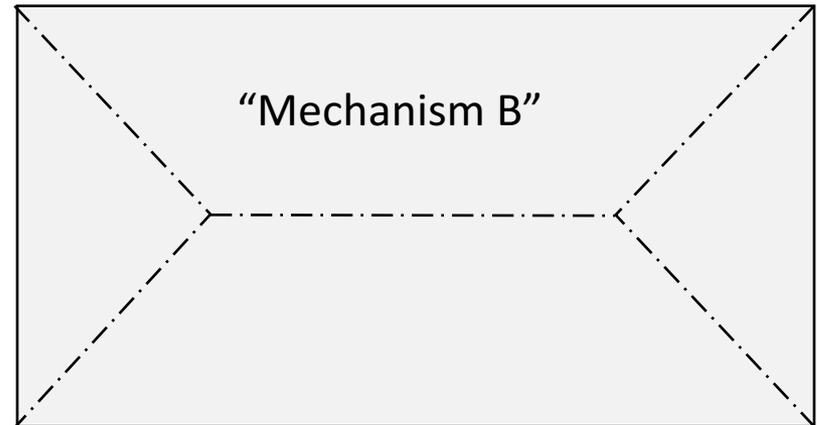


# Increasing deflection of yield-line mechanism

**Yield-line mechanism** is a plastic bending mechanism at small deflections. Yield lines are essentially discrete cracks.

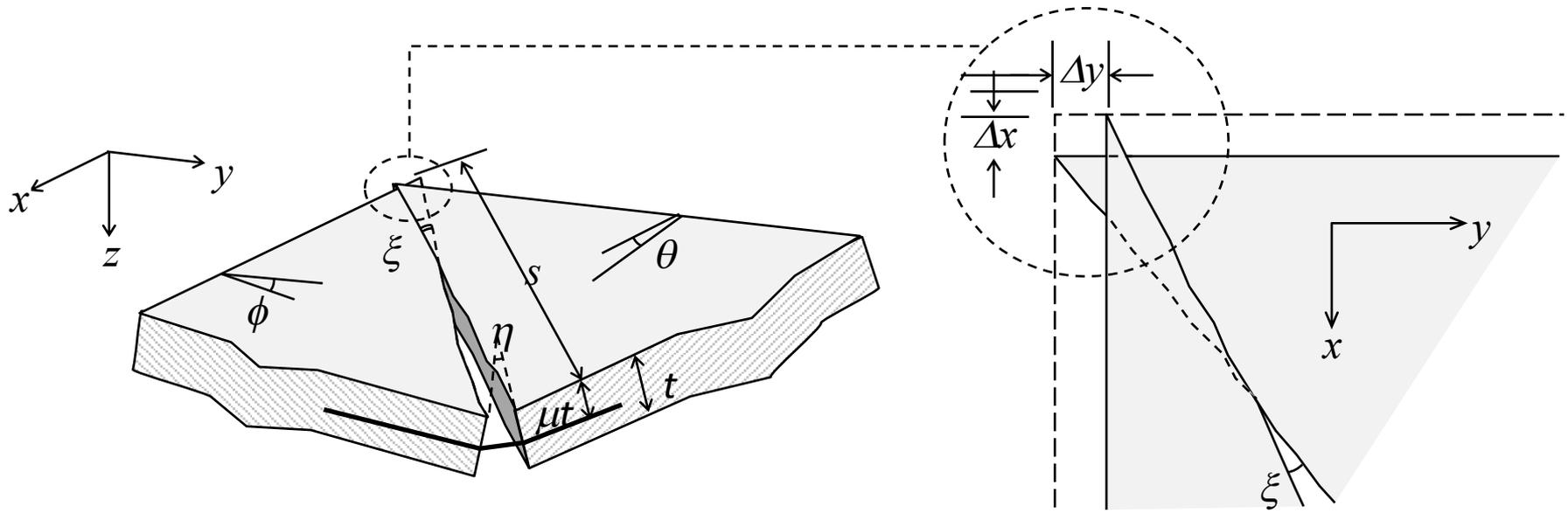
**As deflections start to increase** the yield-line pattern increases the rotations of its flat facets, with the rebar yielding until it fractures.

**So the initial large-deflection mechanism** is this one.





# Geometry of yield-line crack opening



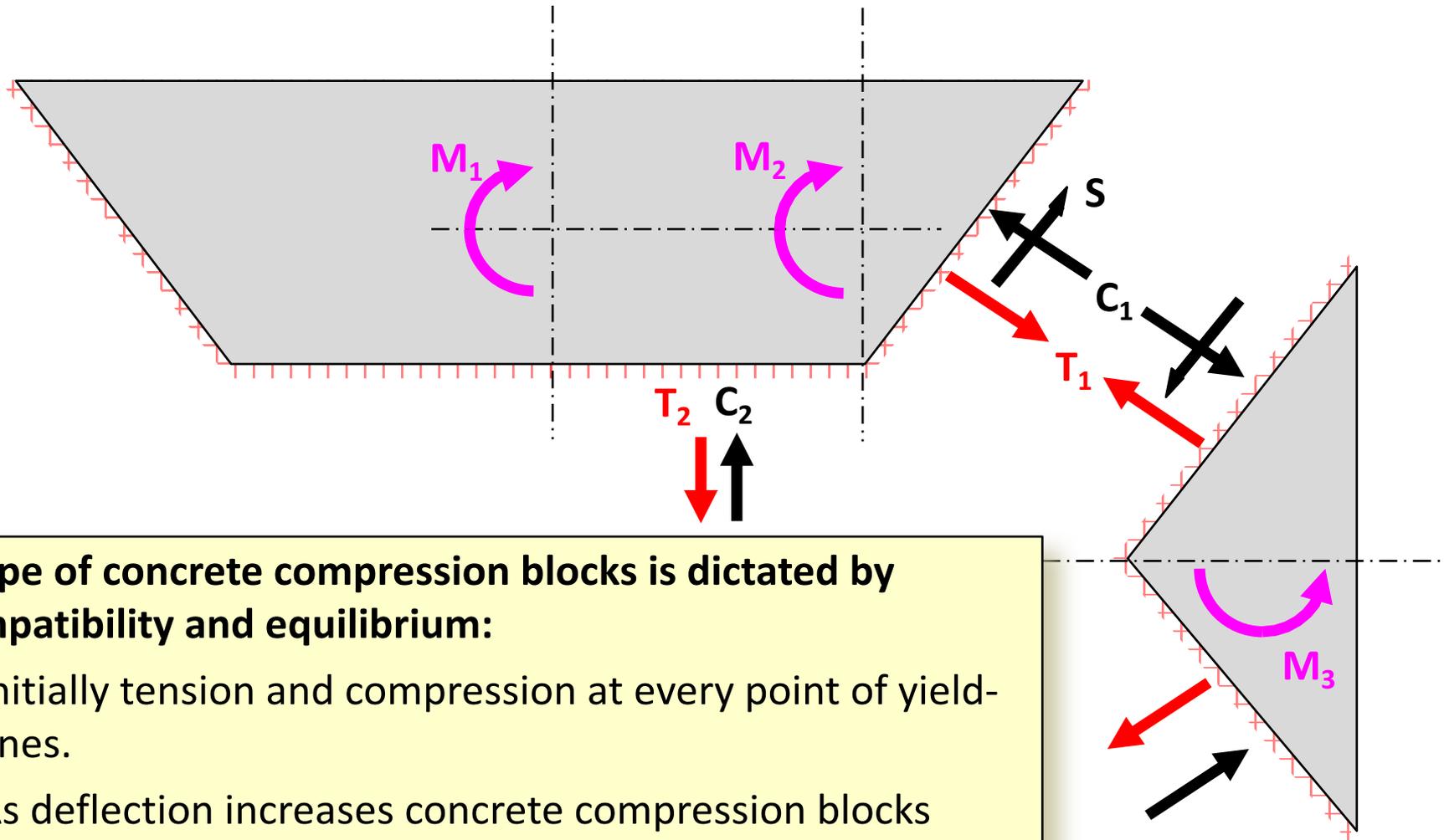
CRACK OPENING AT REBAR LEVEL

TOP SURFACE OF SLAB

**As deflections start to increase the yield-line pattern increases the rotations of its flat facets, and rebar yields across cracks until it fractures.**



# Force equilibrium of Mechanism B

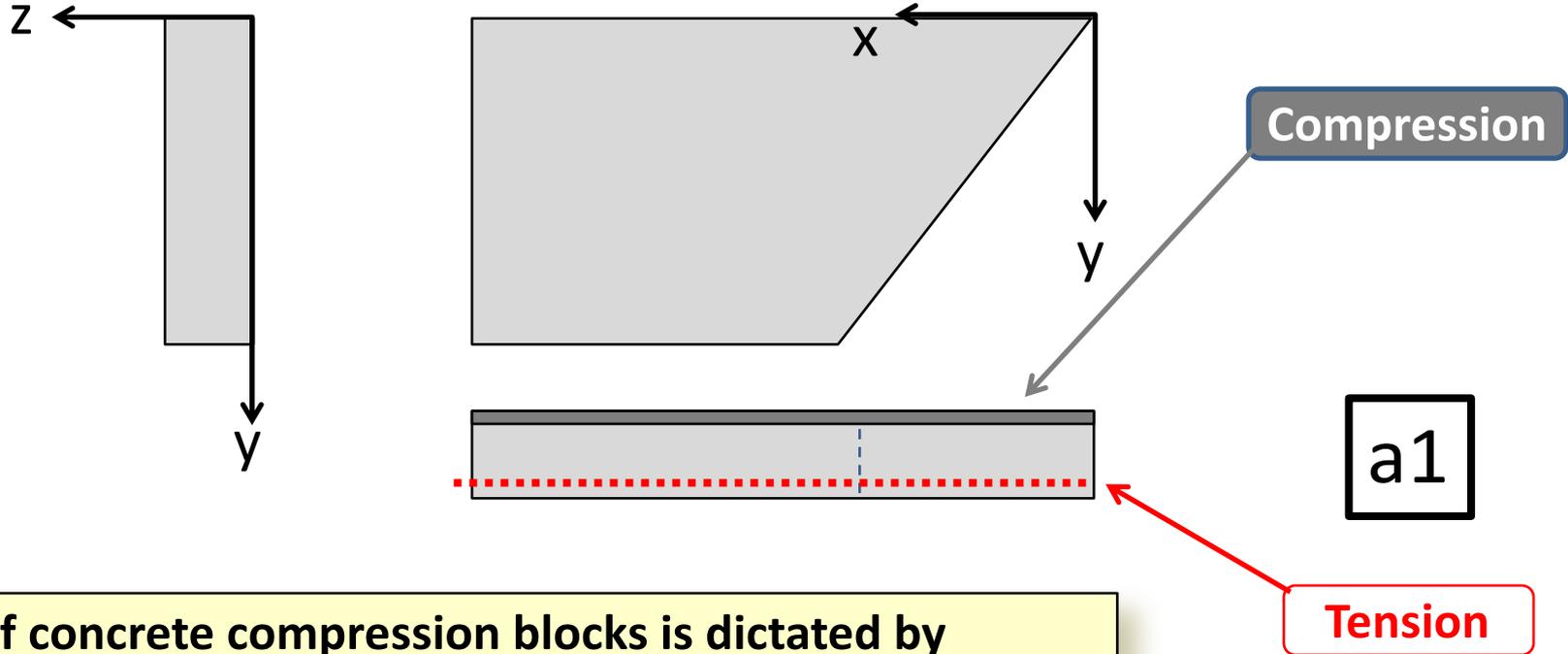


## Shape of concrete compression blocks is dictated by compatibility and equilibrium:

- Initially tension and compression at every point of yield-lines.
- As deflection increases concrete compression blocks concentrate towards slab corners, rebar fractures when its strain exceeds its ductility.
- No tension within compression blocks



# Change of stress blocks – ductile y-reinforcement

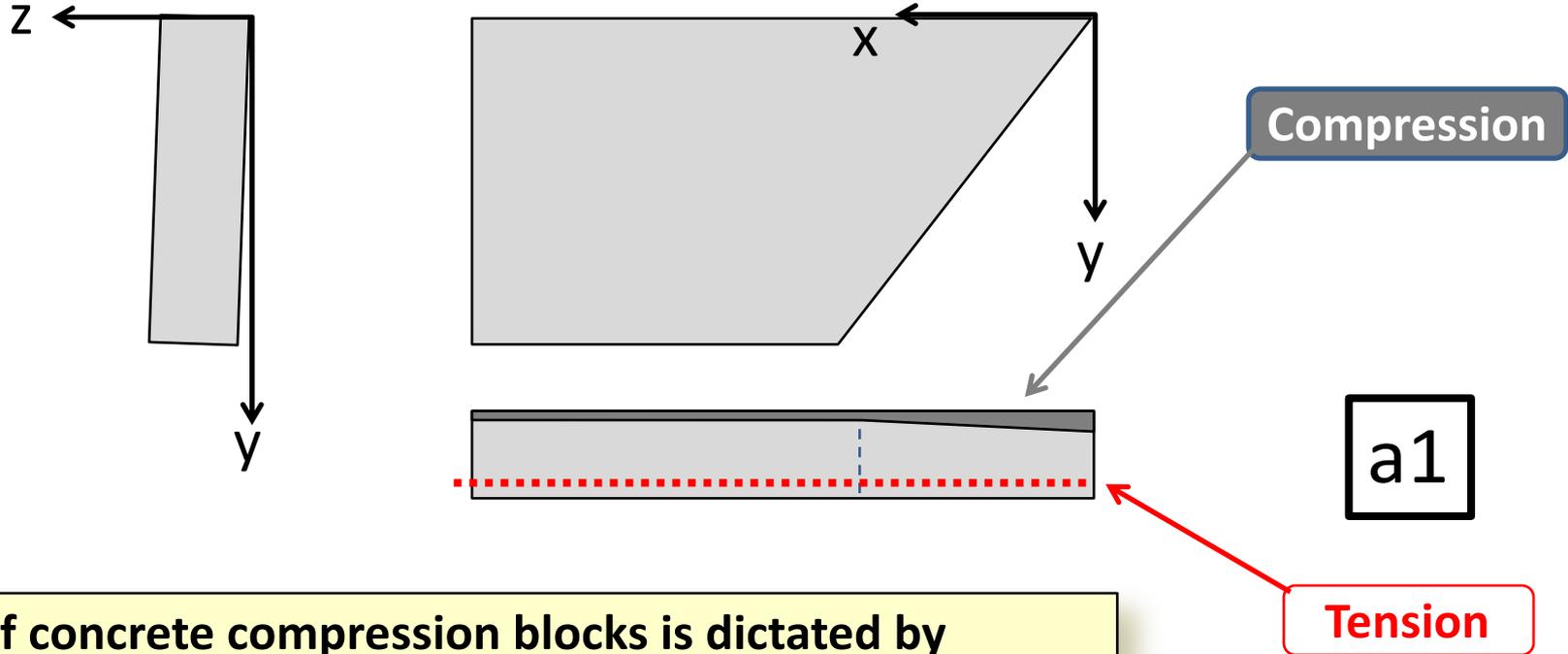


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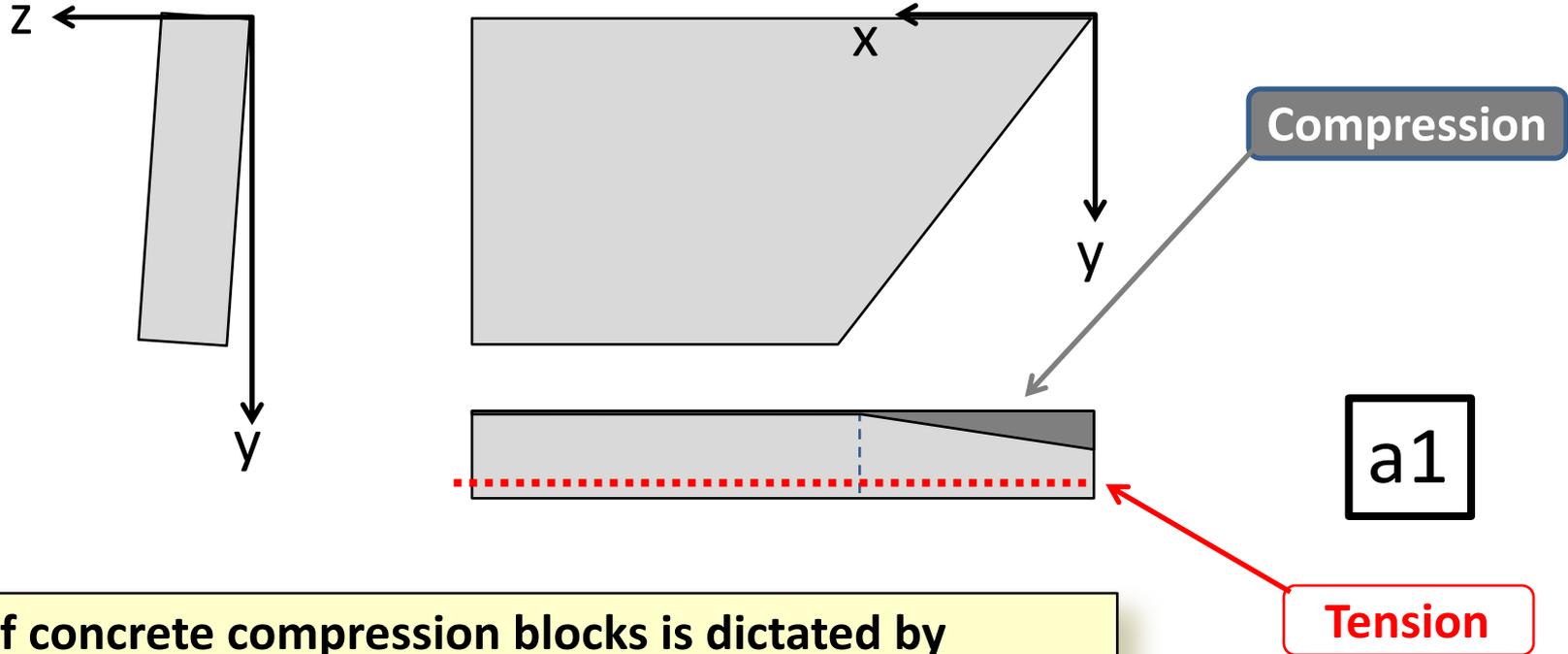


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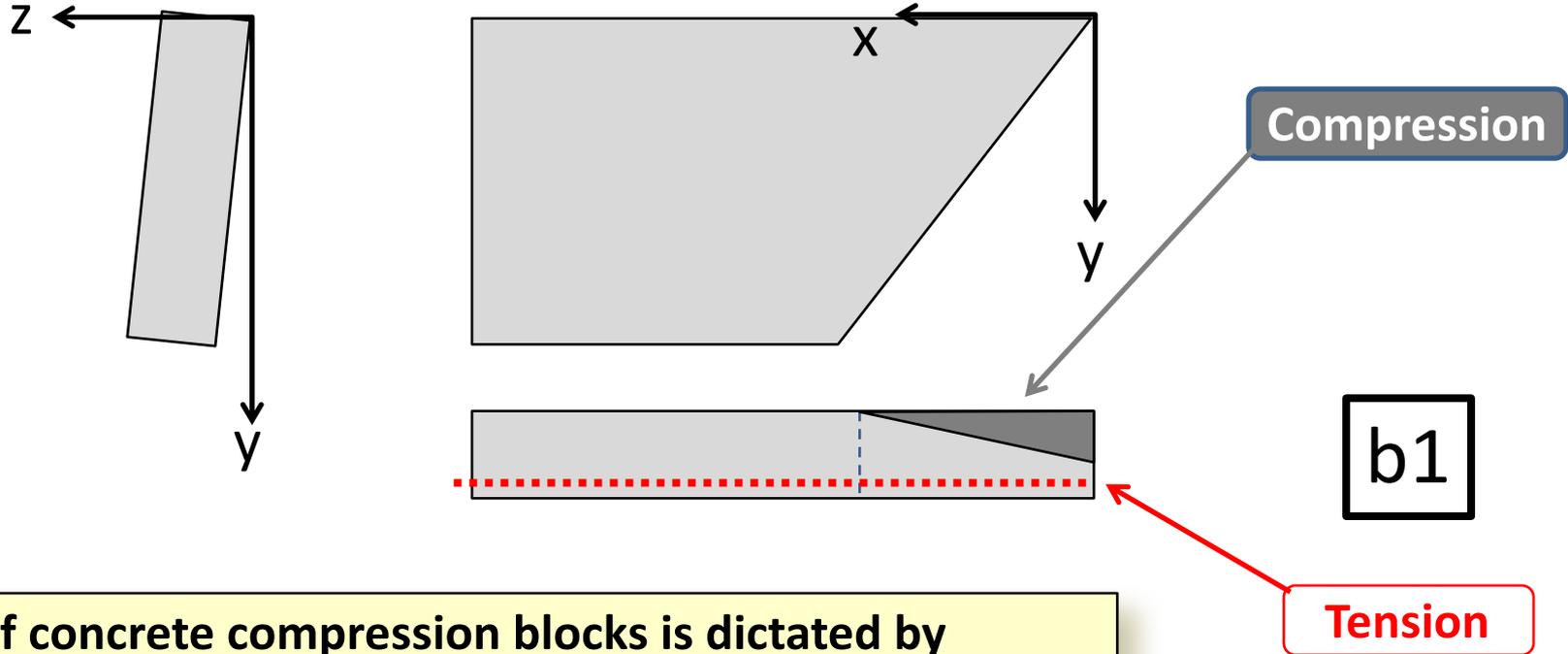


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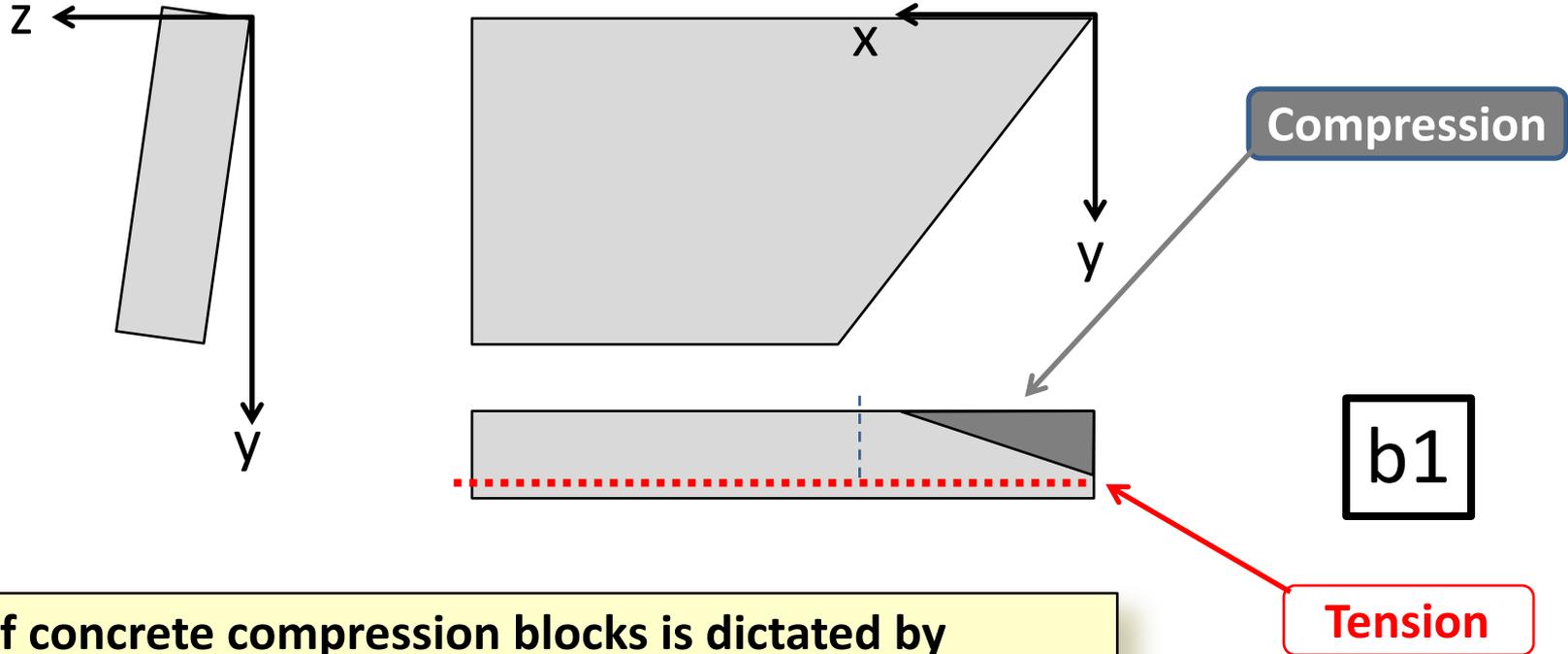


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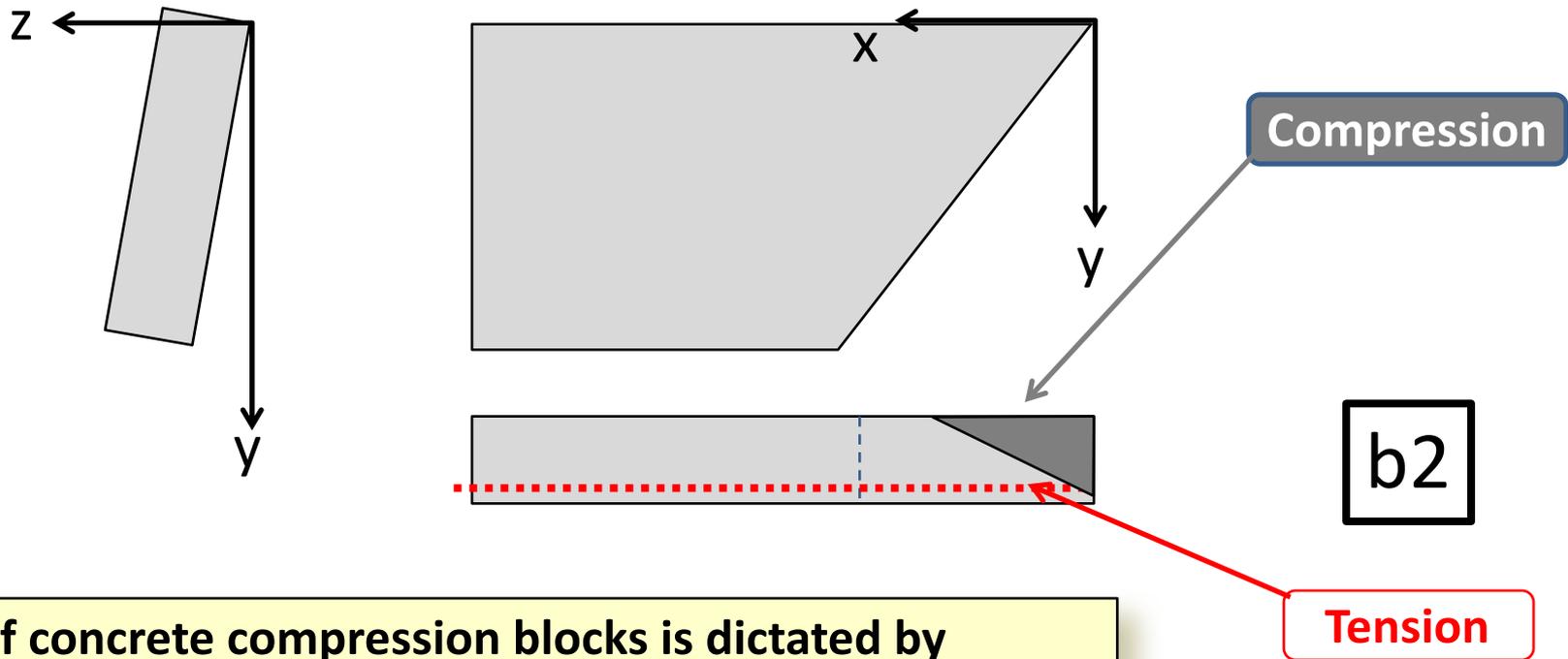


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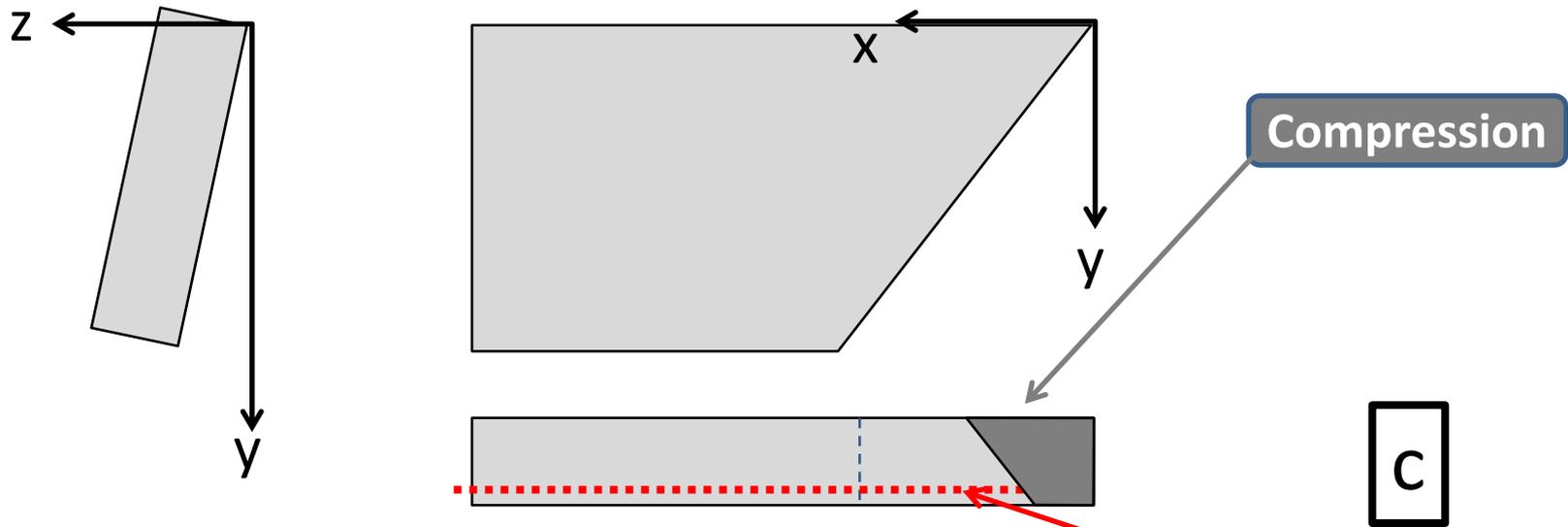


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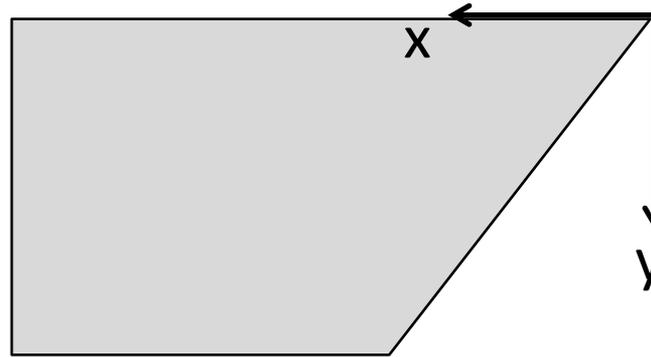
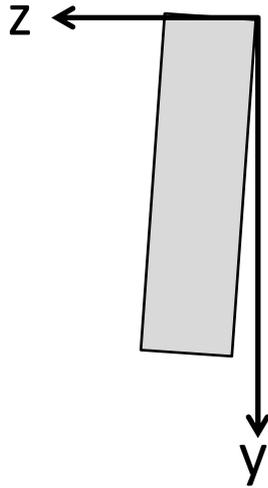


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# Change of stress blocks – possibilities with less ductility



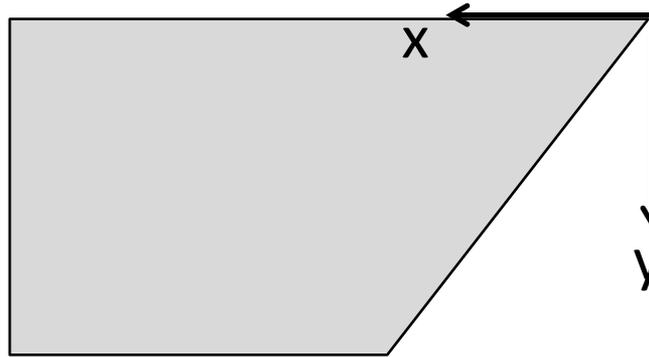
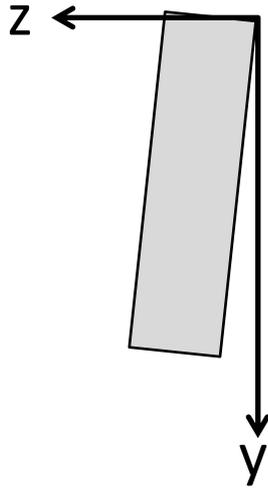
With different rebar ductility, mesh can either fracture abruptly or progressively at any stage.



**Tension**



# Change of stress blocks – possibilities with less ductility



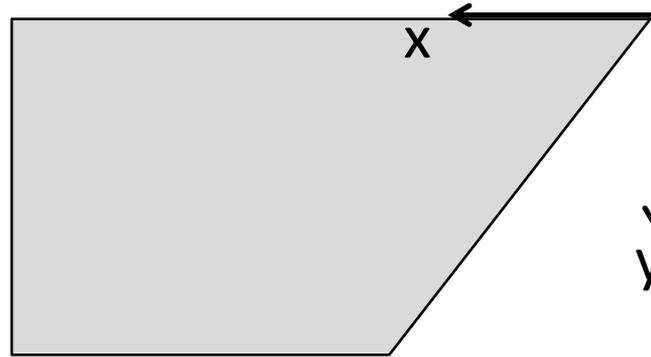
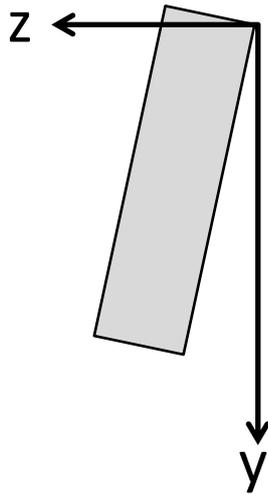
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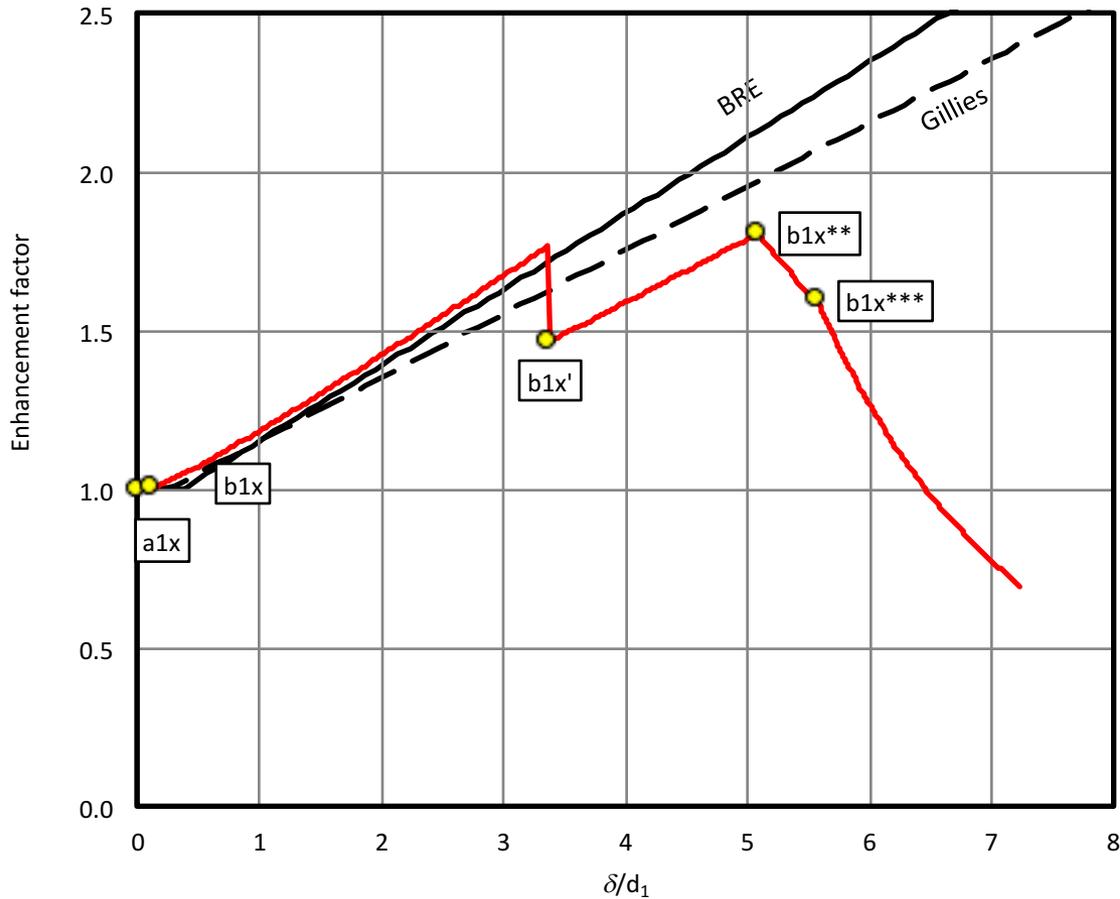


**Tension**

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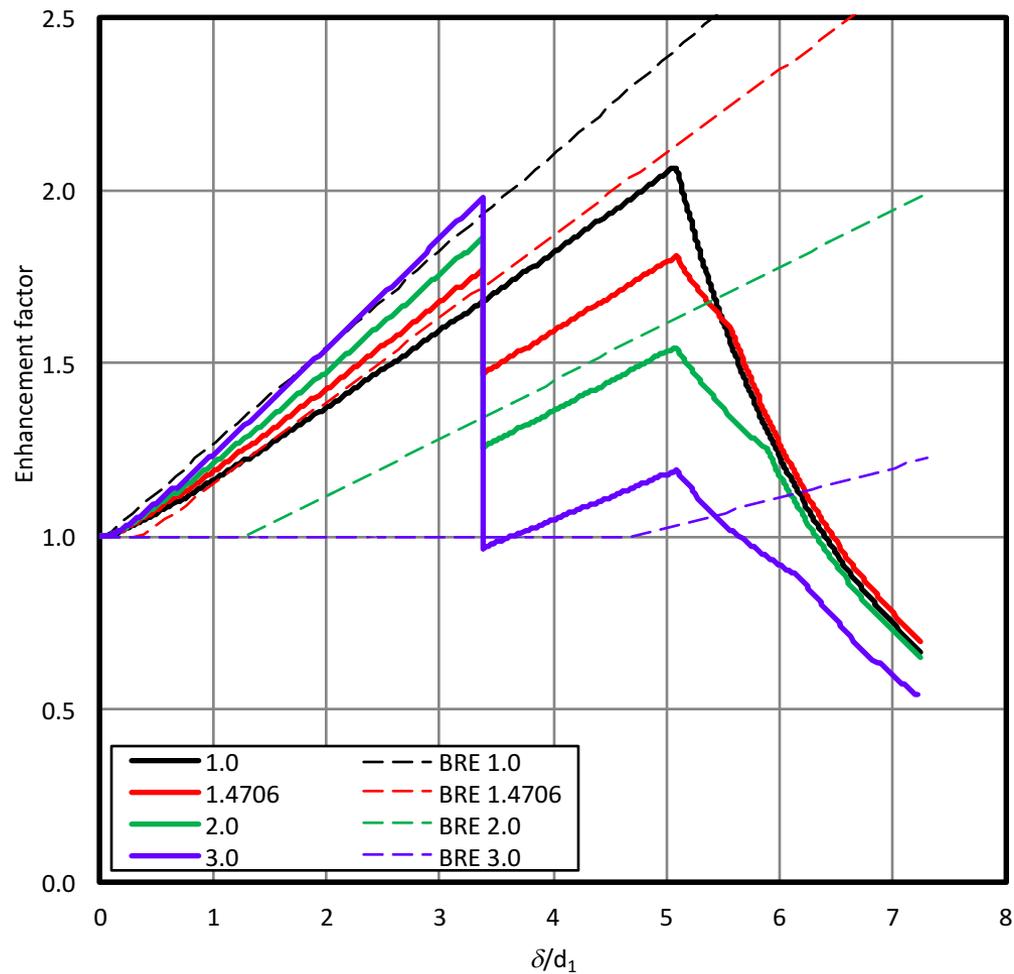
# Garston test comparison



- Slab aspect ratio 1.4706 (6.360m x 9.353).
- 120mm thick, 52MPa concrete;
- Edges vertically supported.
- A142 mesh (142 mm<sup>2</sup> per metre, 580MPa steel at 200mm spacing in x and y directions) at 69mm effective depth;
- Mesh ductility 12%.

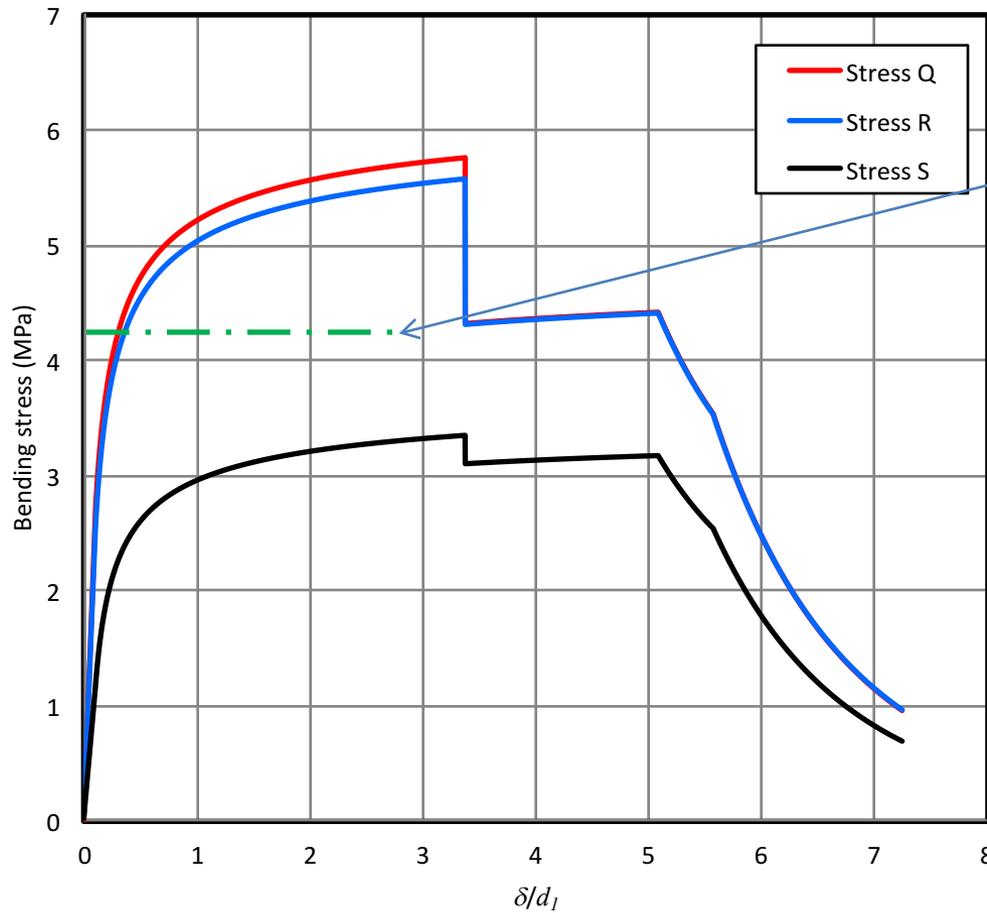


# Garston comparison for different aspect ratios

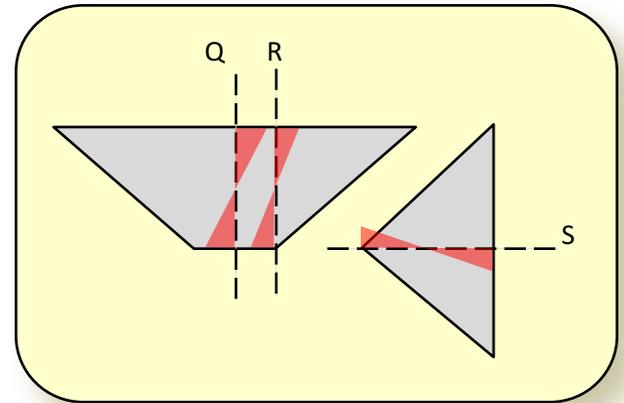




# Garston – apply tensile strength to change mechanism

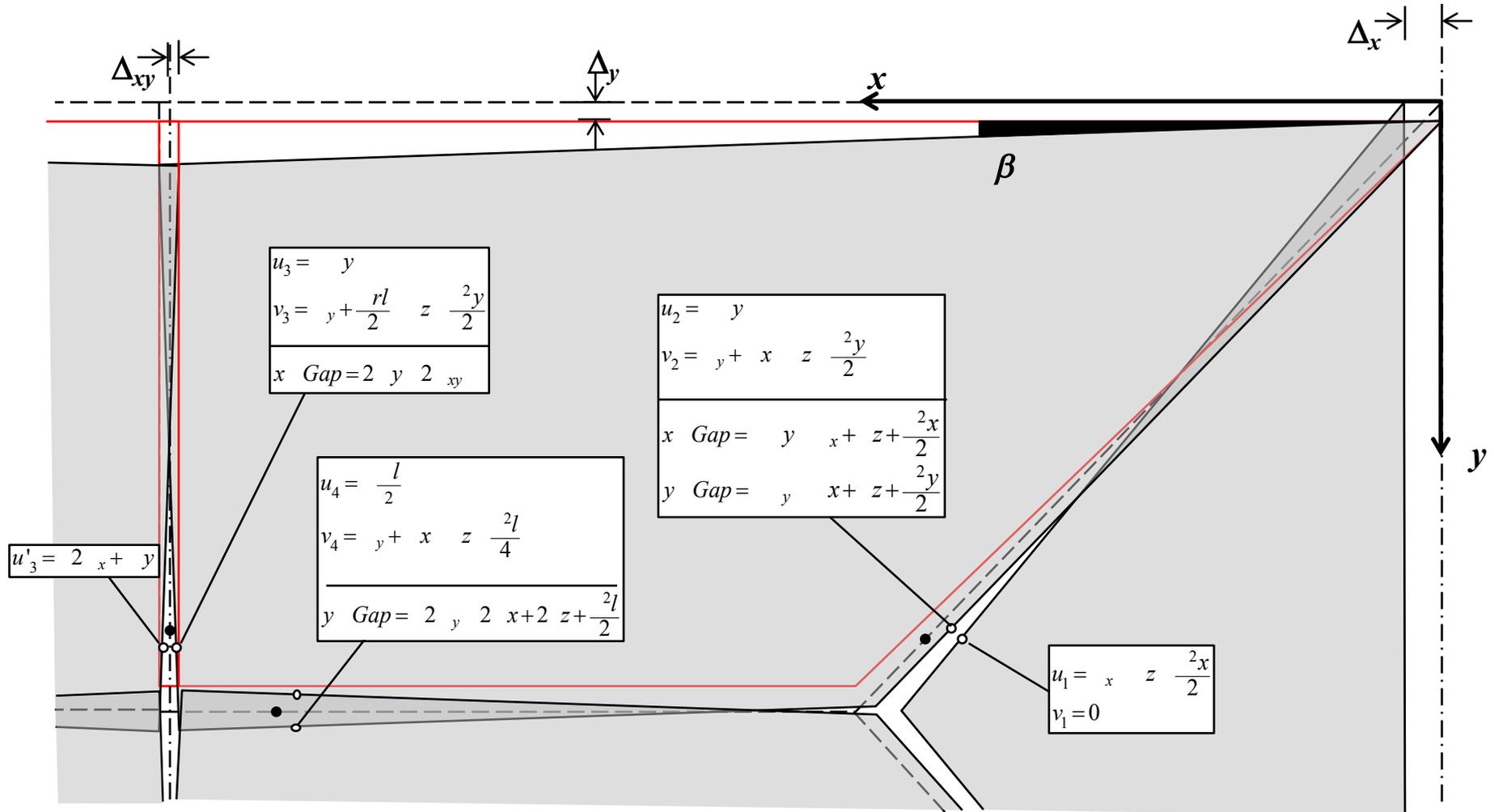


EC2 tensile strength for C52  
 $[0.3f_c^{0.67}]$



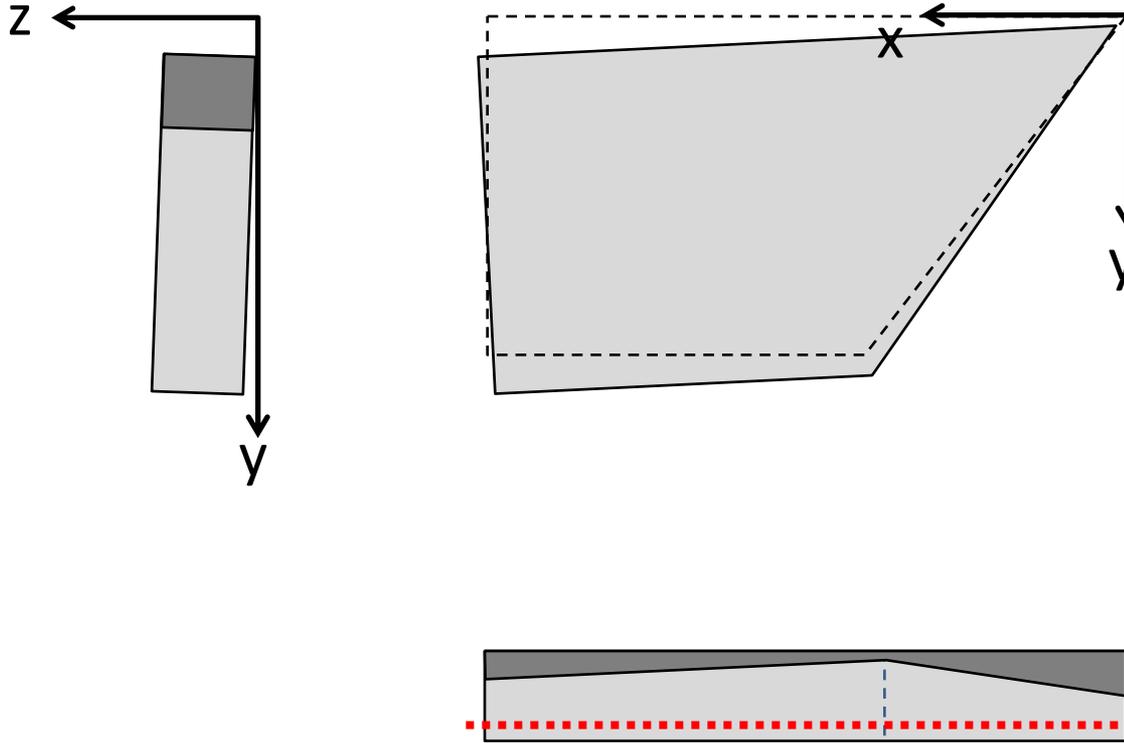


# New mechanism – central through-depth crack



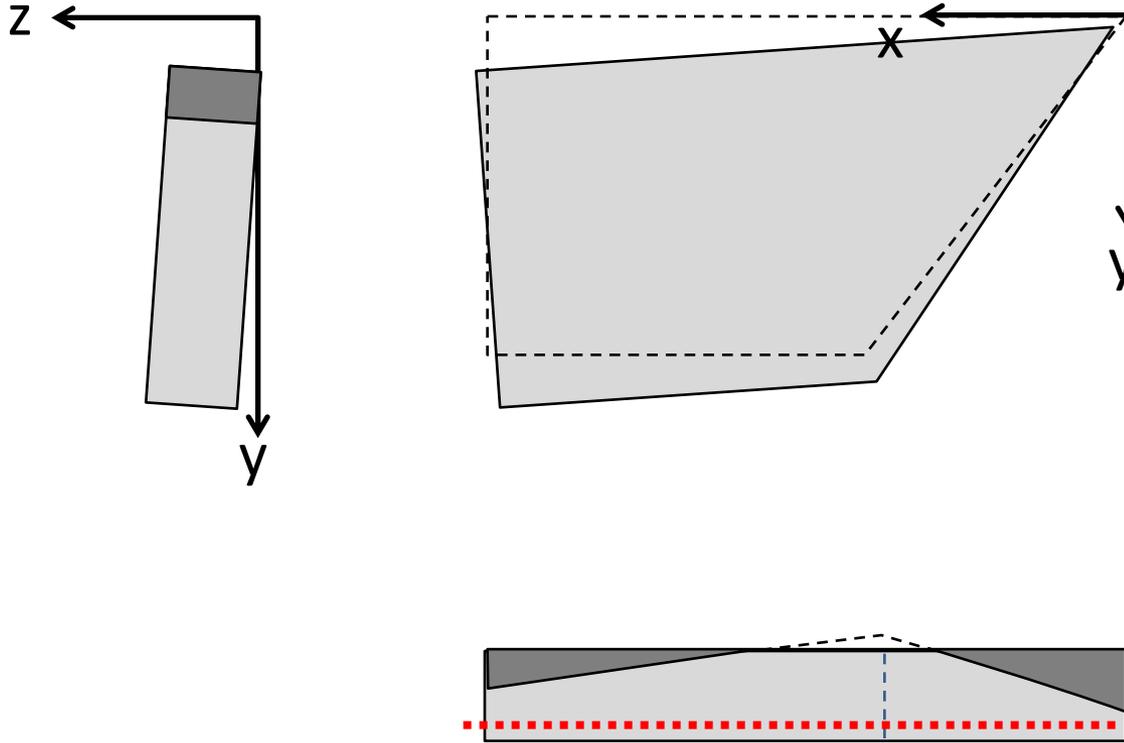


# Change of stress blocks – ductile y-reinforcement



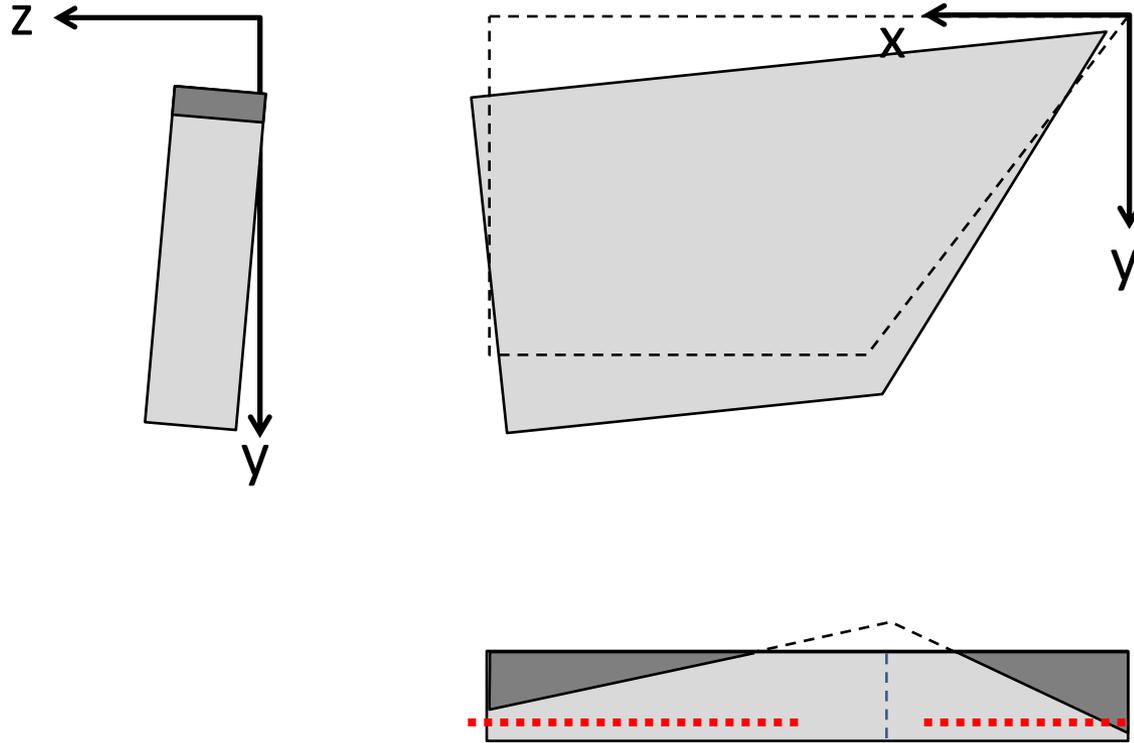


# Change of stress blocks – ductile $\gamma$ -reinforcement



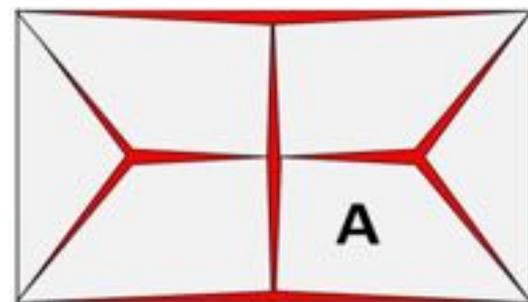
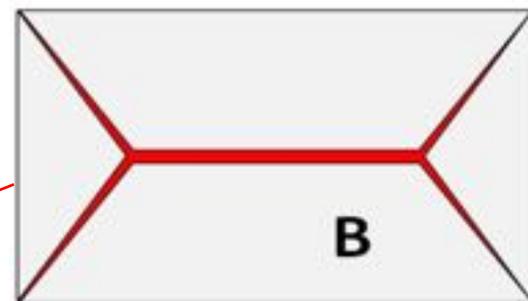
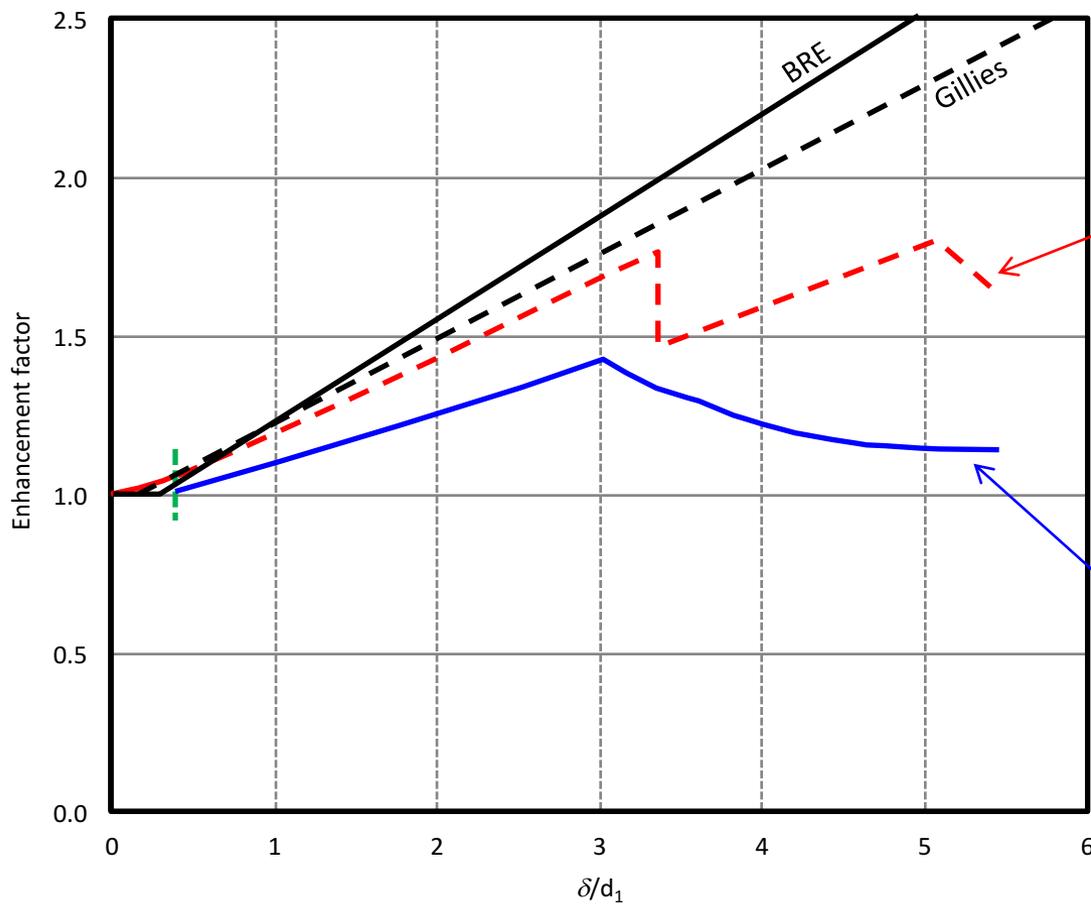


# Change of stress blocks – ductile $y$ -reinforcement





# From basic mechanism to centrally cracked



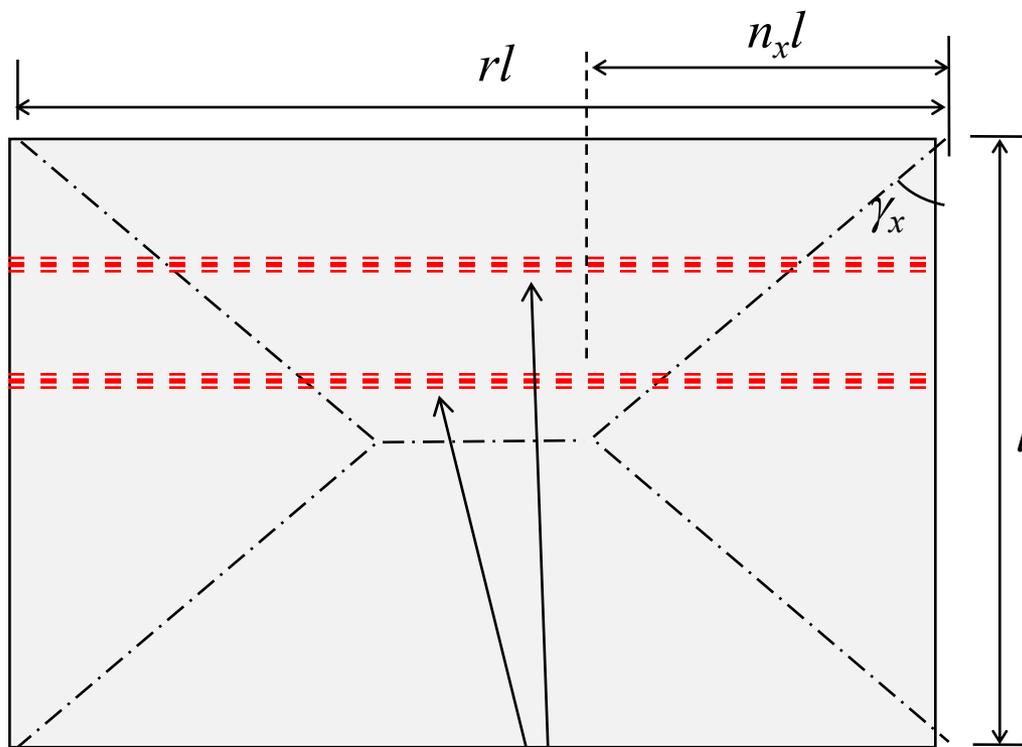


**With attached steel  
beams ...**

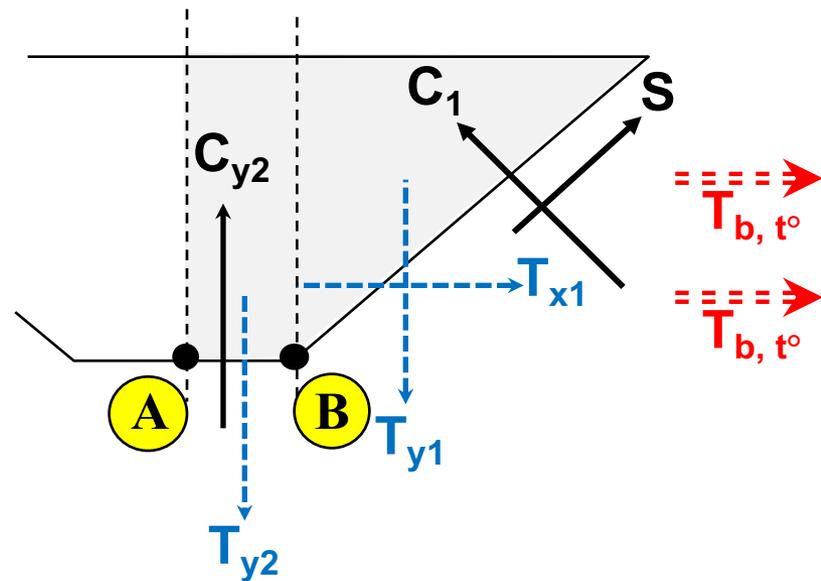
**... the yield-line mechanism changes.**



# Forces on the $x$ -aligned mechanism

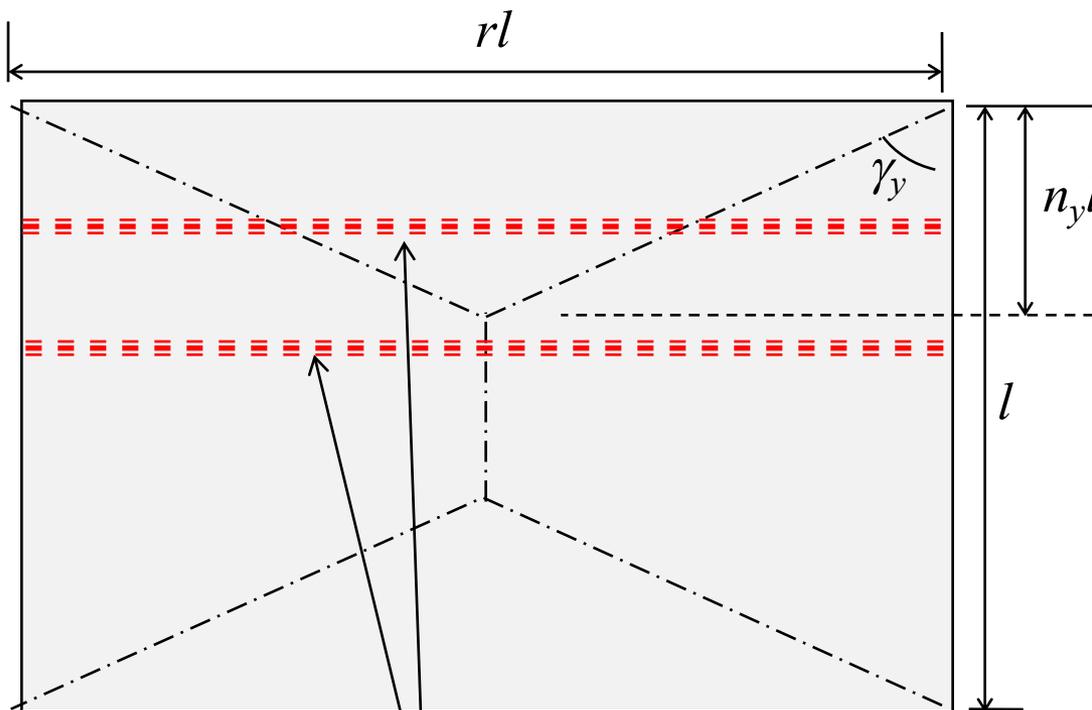


Unprotected beams at high temperature

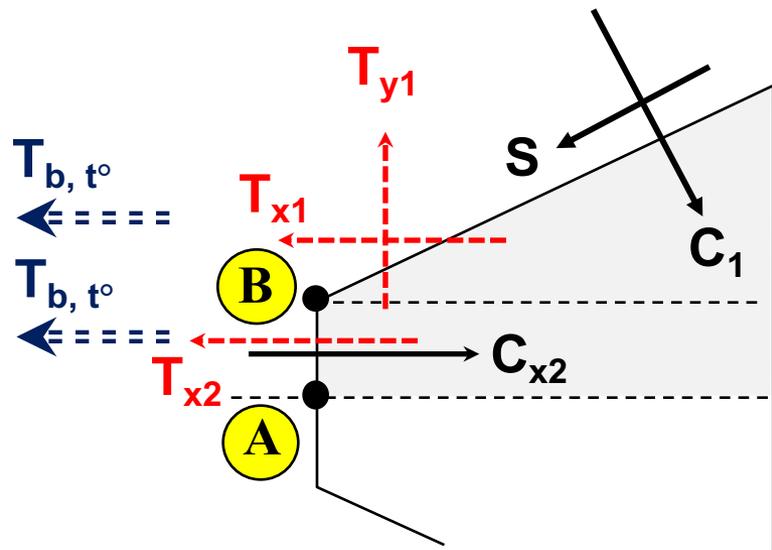




# Forces on the $y$ -aligned mechanism

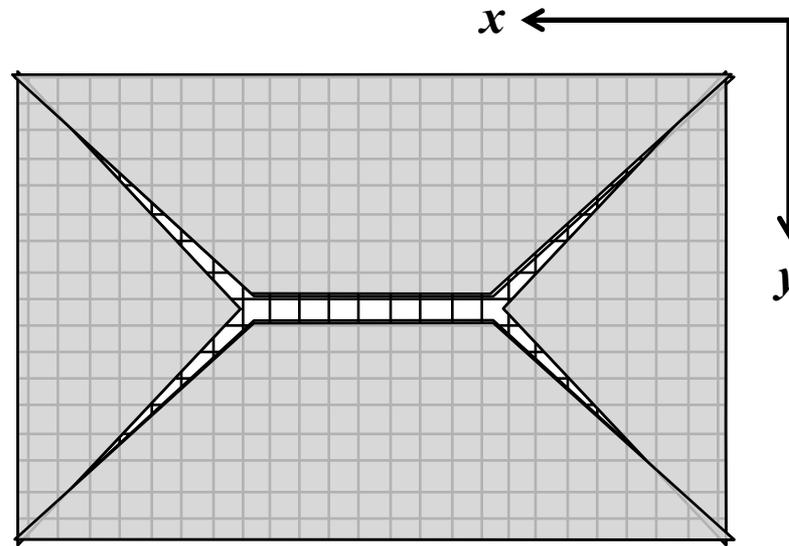


Unprotected beams at high temperature





# Combinations of compression block and rebar fracture

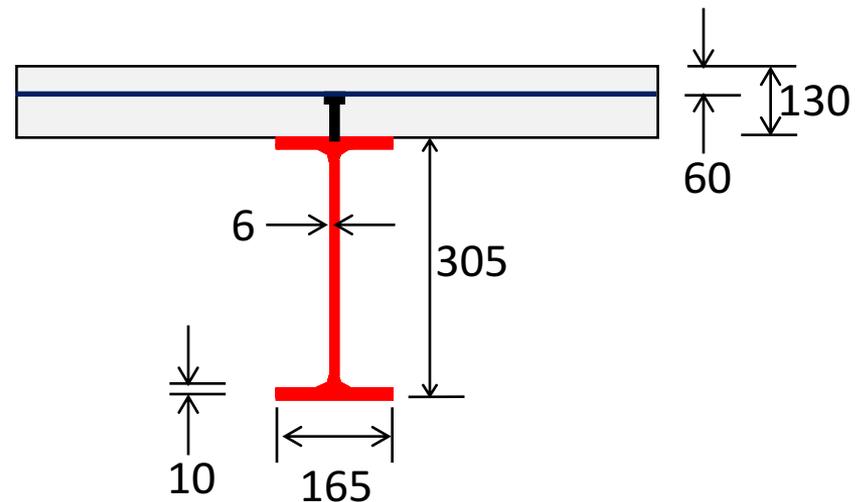
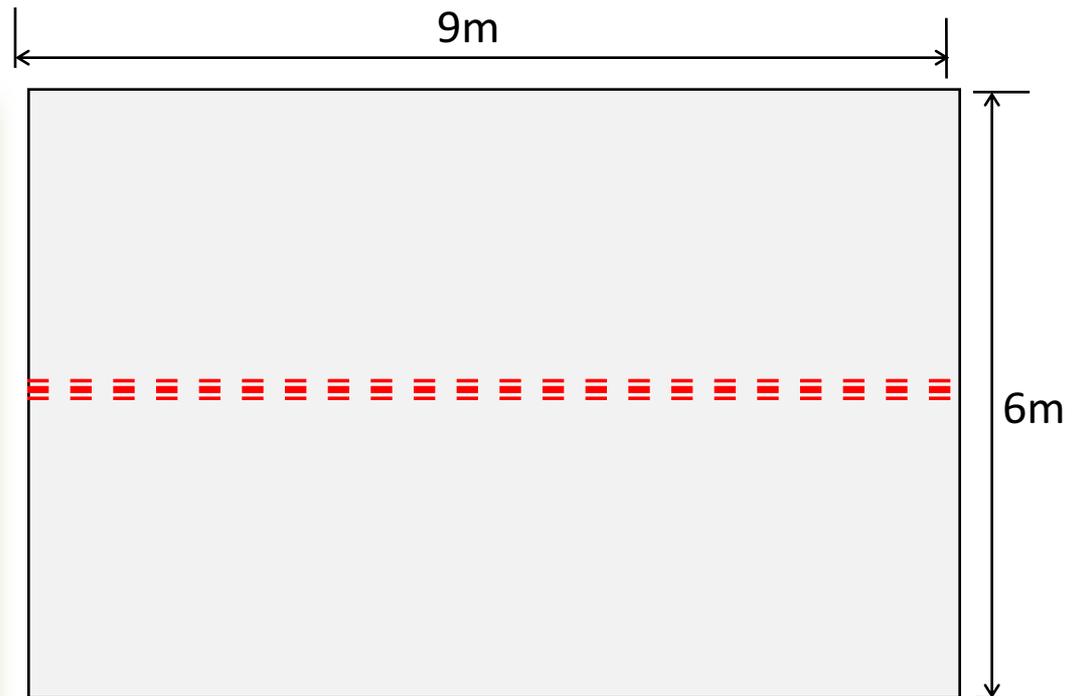


Compression block		Reinforcement mesh fracture level ( <i>x</i> -aligned mechanism)					
		None	Central <i>y</i>	Diag. <i>x</i>	Central + Diag. <i>y</i>	Central + Diag. <i>x</i>	Central + Diag. <i>x, y</i>
Full	above mesh	a1	a1'	a1*	a1**	a1*'	a1***
	below mesh	a2	a2'	a2*	a2**	a2*'	a2***
Triangular	above mesh	b1	b1'	b1*	b1**	b1*'	b1***
	below mesh	b2	b2'	b2*	b2**	b2*'	b2***
Trapezoidal		c1	c1'	c1*	c1**	c1*'	c1***



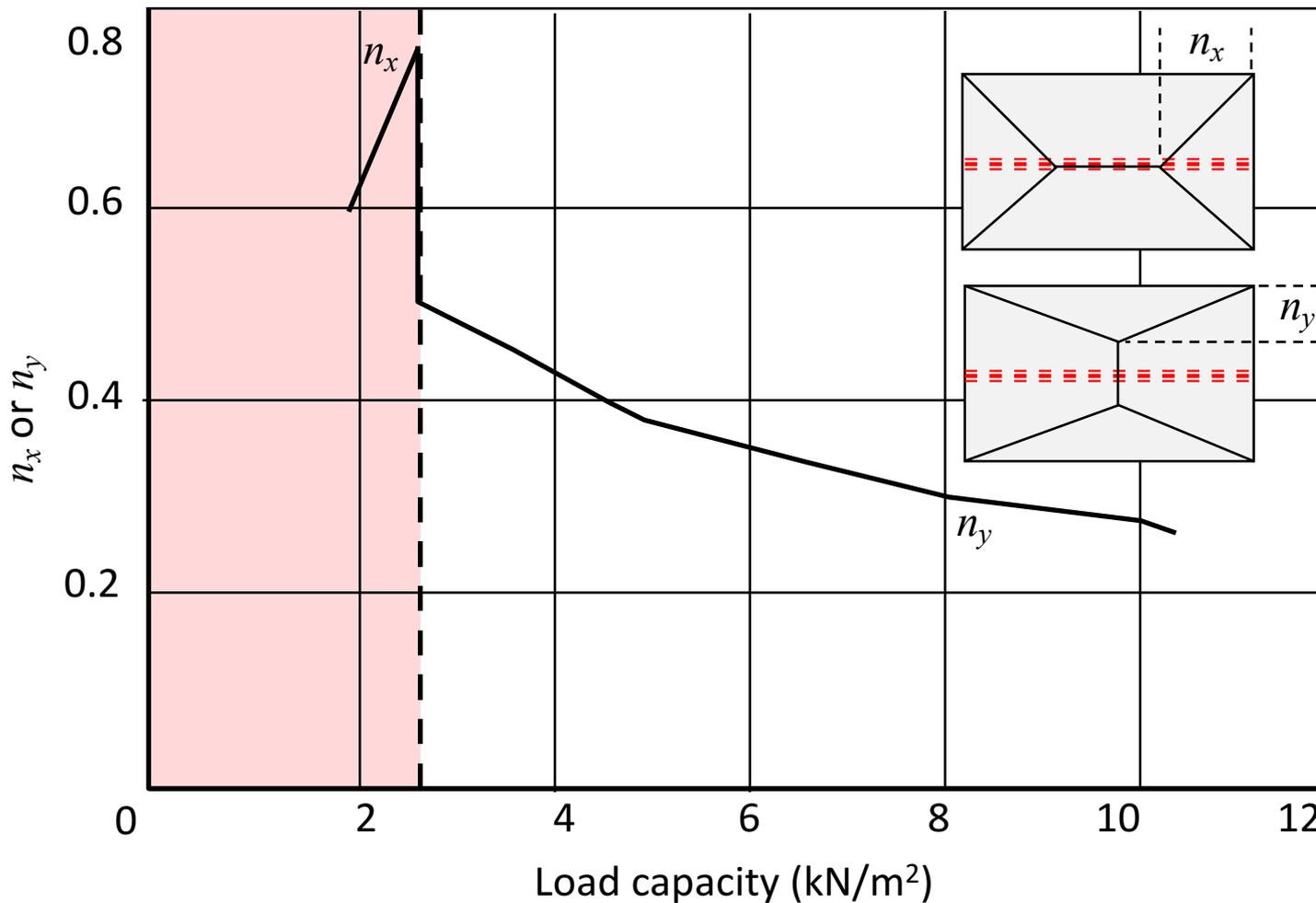
# Example of application: 9m x 6m composite slab

- 130mm thick slab, 30MPa concrete;
- A142 mesh (142 mm<sup>2</sup> per metre, 500MPa steel at 200mm spacing in x and y directions) at 60mm effective depth;
- Mesh effective ductility (over 200mm length) 1%: fracture crack-width 2mm;
- One central downstand steel beam, 305x165UKB40, Grade S275 - unprotected against fire;
- Edges vertically supported.



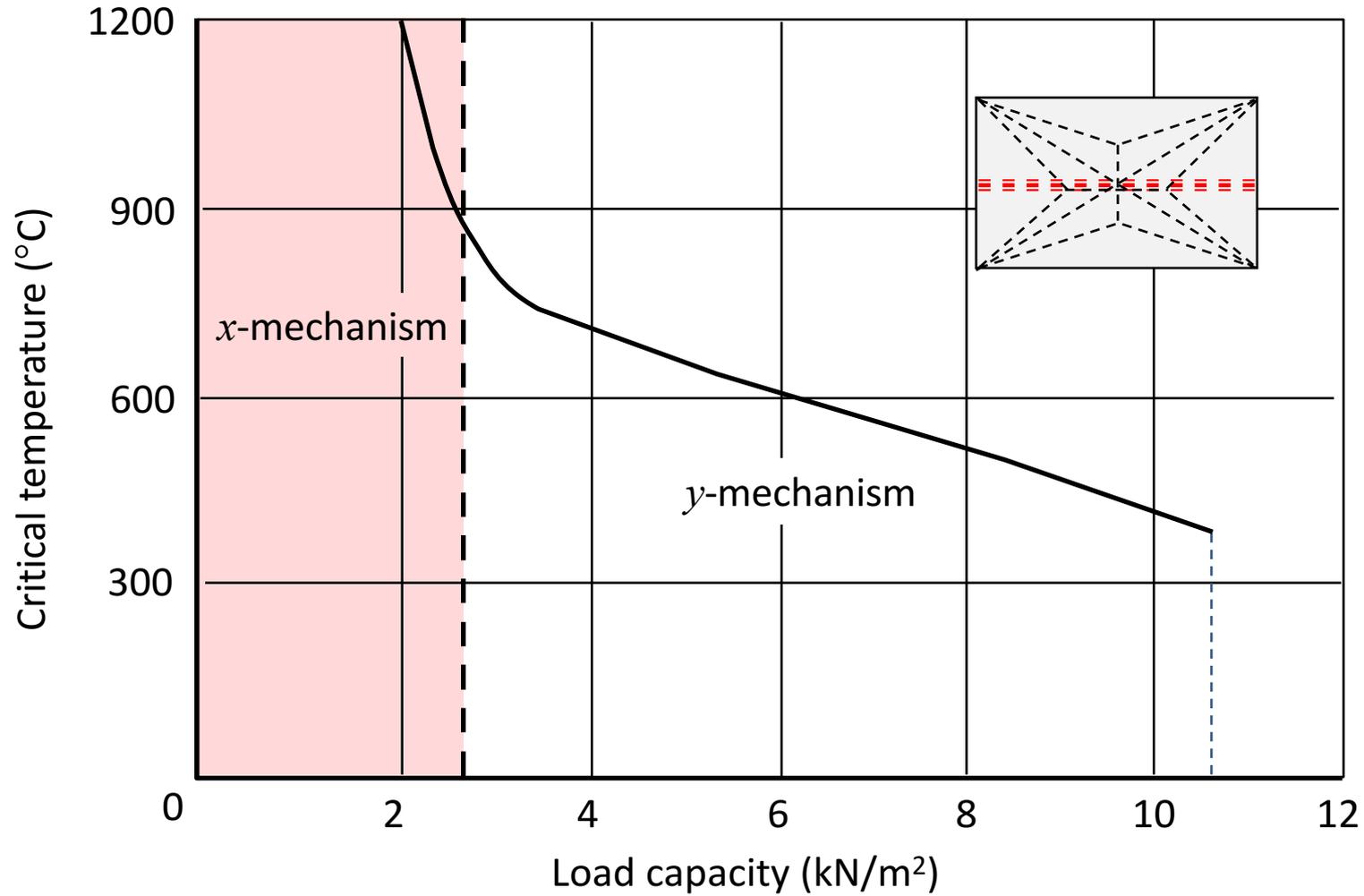


# Initial yield-line parameter for different load capacities



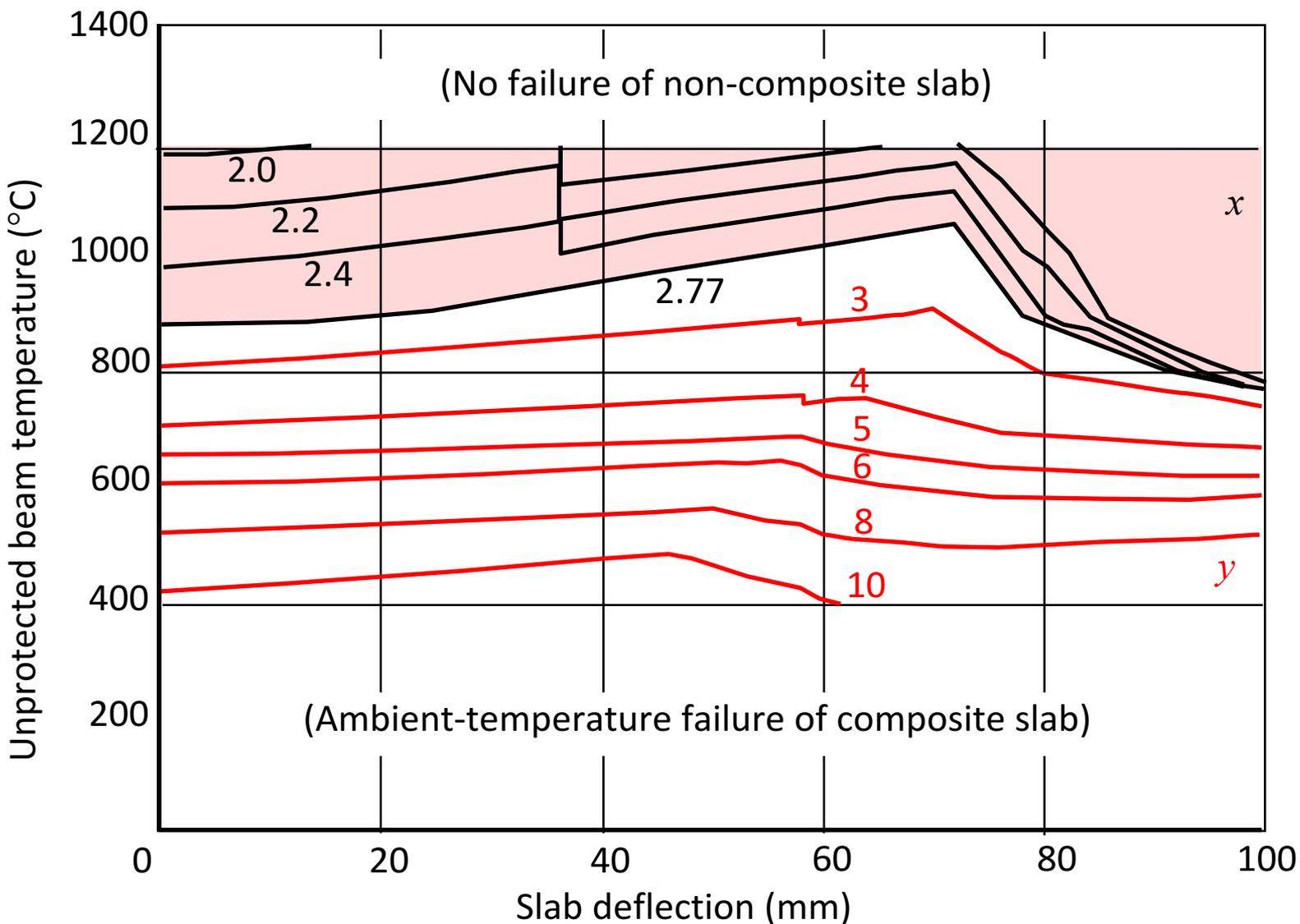


# Critical temperature variation with load capacity



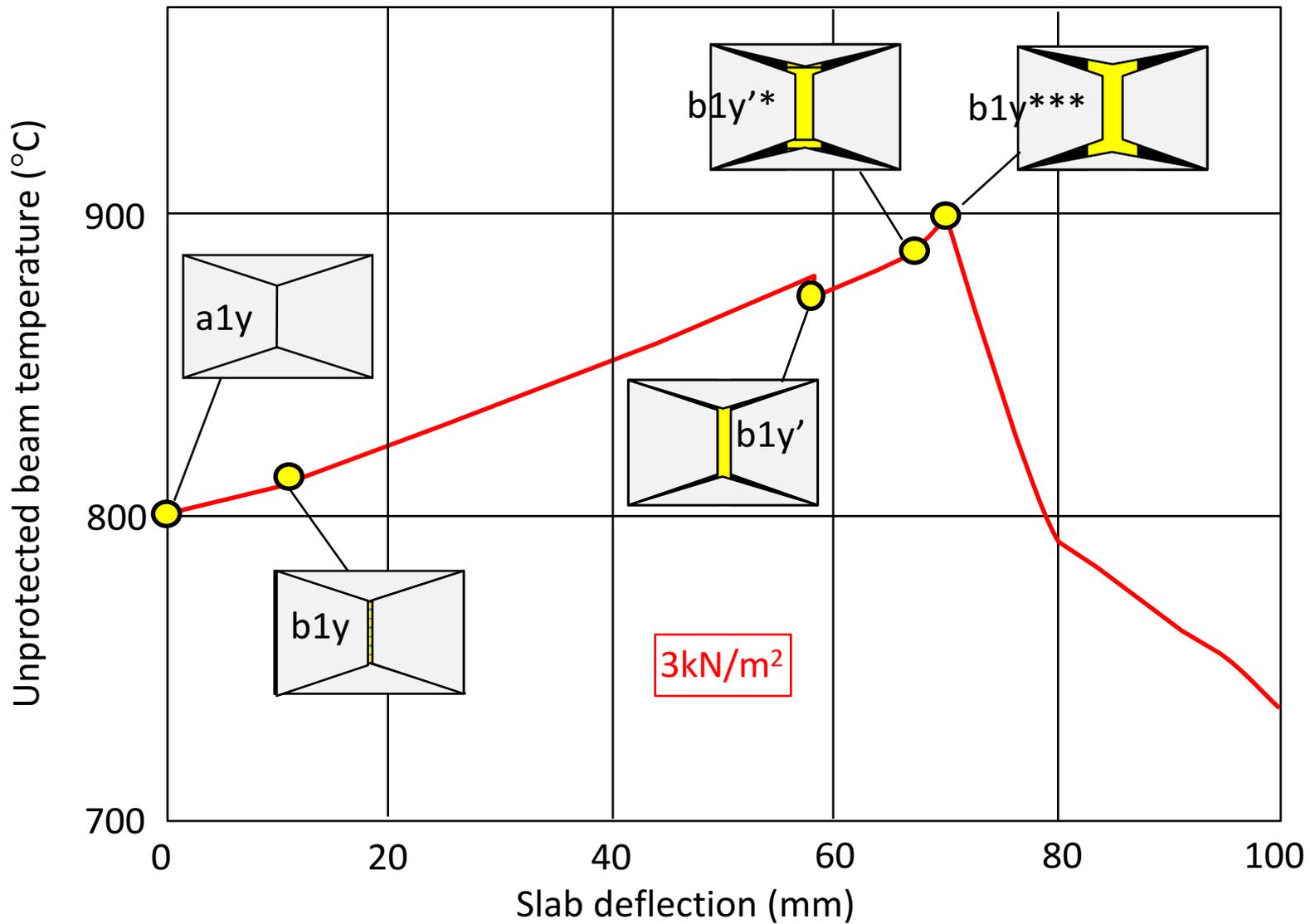


# Enhancement of critical steel temperature with deflection



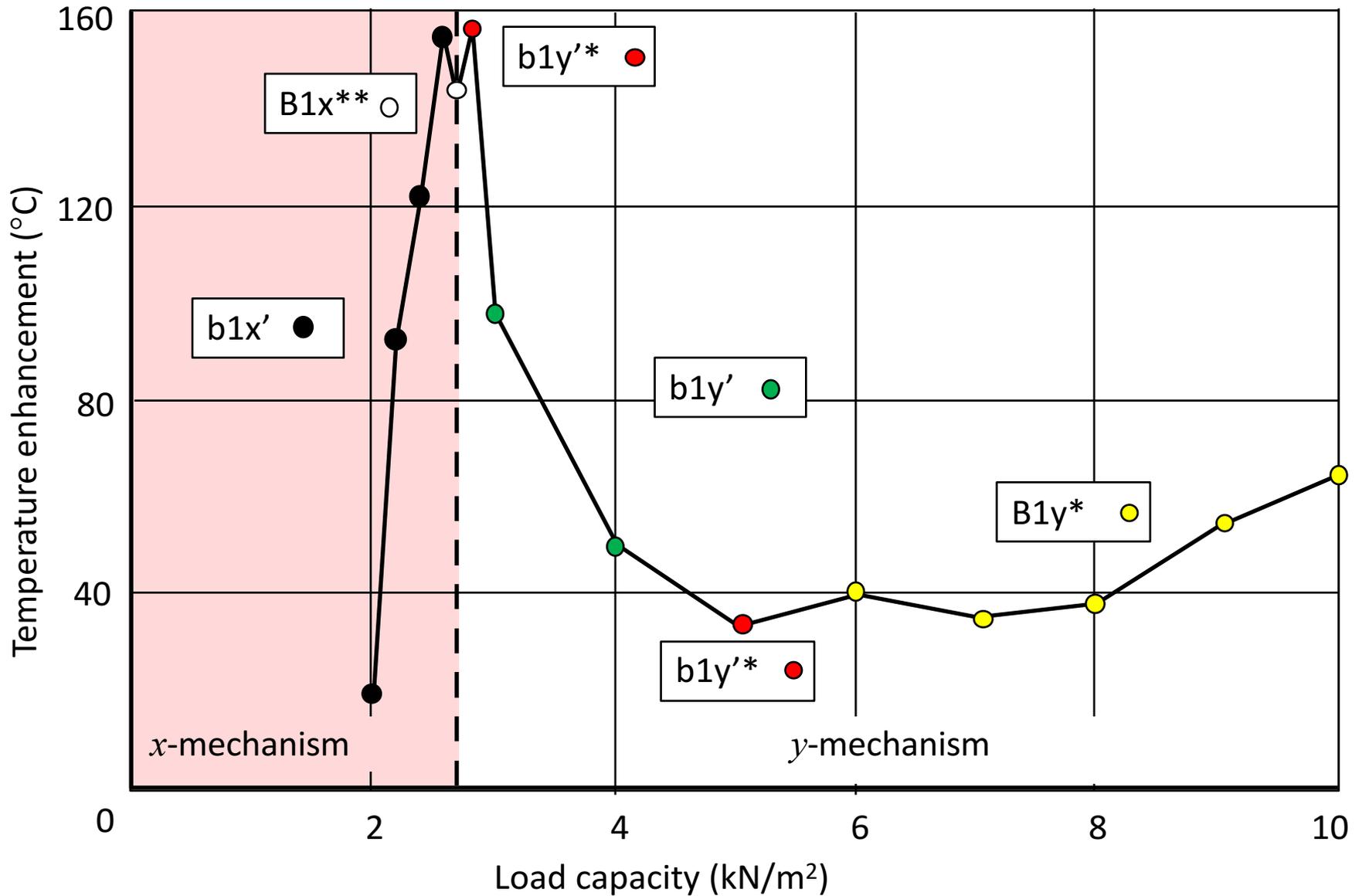


# Enhancement of critical steel temperature with deflection



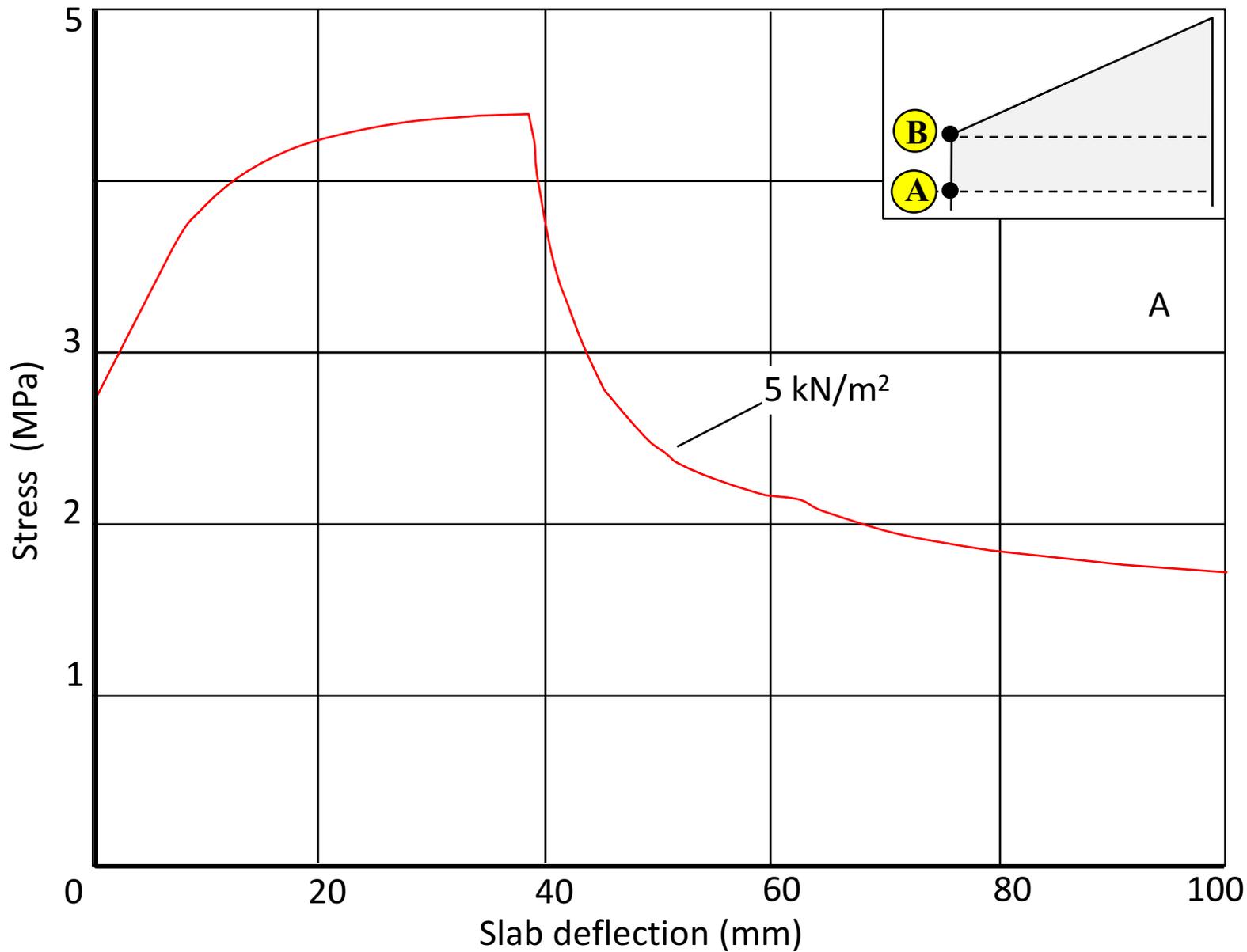


# Maximum steel temperature enhancements





# Maximum tensile stress at section A





### **Existing simplified methods:**

#### For concrete slabs:

- Fixed membrane traction distribution – independent of slab deflection.
- Membrane traction distribution only valid while concrete has compression along whole yield lines.
- Assumes central crack fully formed. Rebar at ultimate strength (+10%)
- Enhancement factor starts below 1.0.

#### For composite slabs in fire:

- Yield-line pattern based on non-composite slab.
- Superposes high-temperature composite beam capacity and deflection-controlled slab enhancement.
- Criterion for mid-span through-depth crack is meaningless.



## **The new approach:**

### For all slabs:

- Based on the kinematics of deflecting flat facets of the small-deflection yield line mechanism, together with in-plane equilibrium of the concrete and steel forces.
- Allows concrete stress blocks to move and mesh to fracture across yield lines.

### For composite slabs in fire:

- Keeps load constant, allows beams temperature to increase until yield line mechanism forms.
- Enhancement of steel beam temperature with deflection.

## **Biggest problems to be solved:**

- Fracture ductility of rebar across discrete cracks - yield lines or through-depth mid-span crack.
- Concrete tensile stress to initiate the mid-span (or intersection) crack.



**Thank you**