Tensile Membrane Action of Composite Slabs in Fire

Are the current methods really OK?

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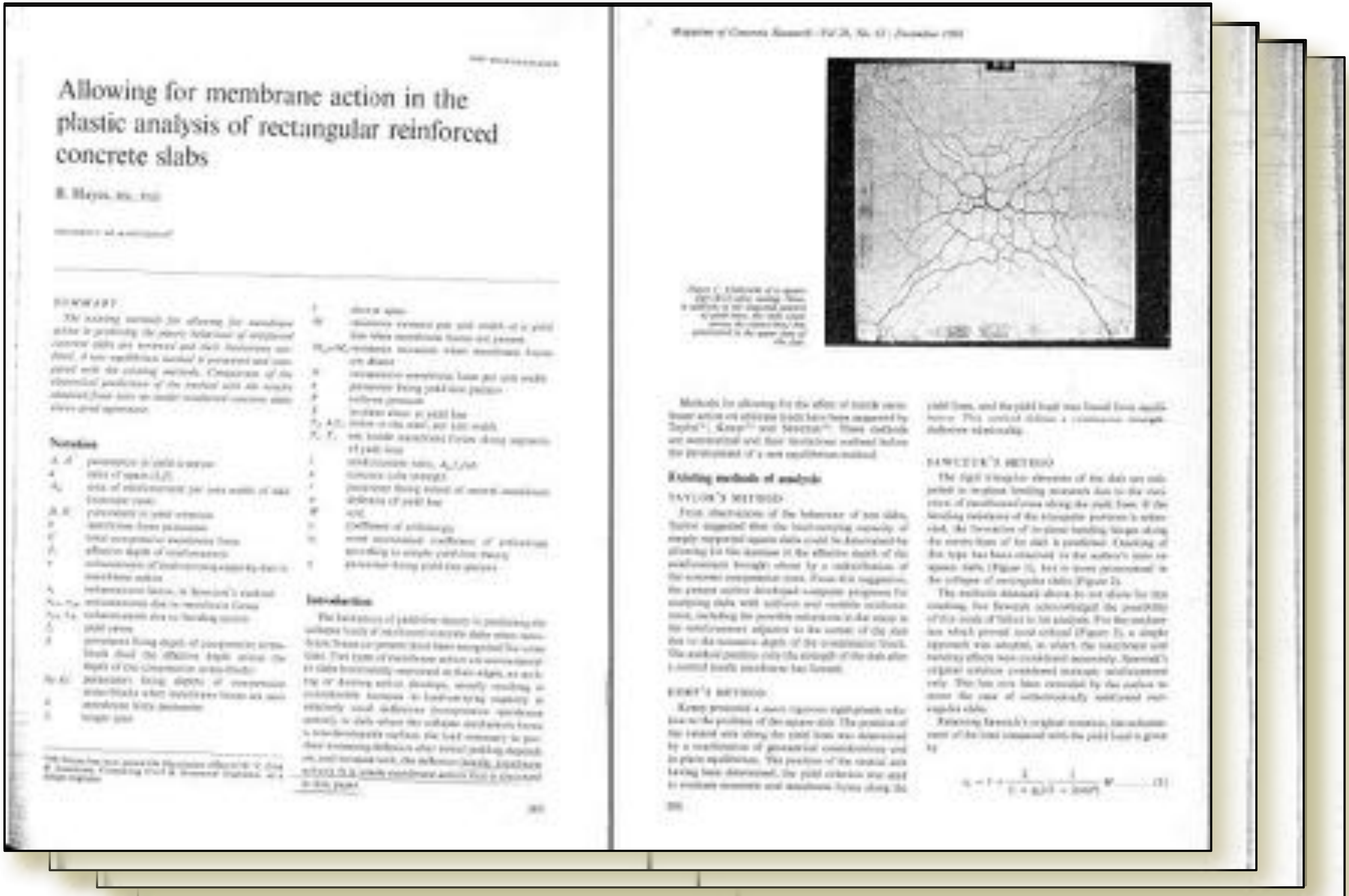
Cardington

Max beam temperature ~1150°C

cf. Code critical temperature ~ 680°C

The basis of all current simplified methods: Hayes (1968)


Allowing for membrane action in the plastic analysis of rectangular reinforced concrete slabs

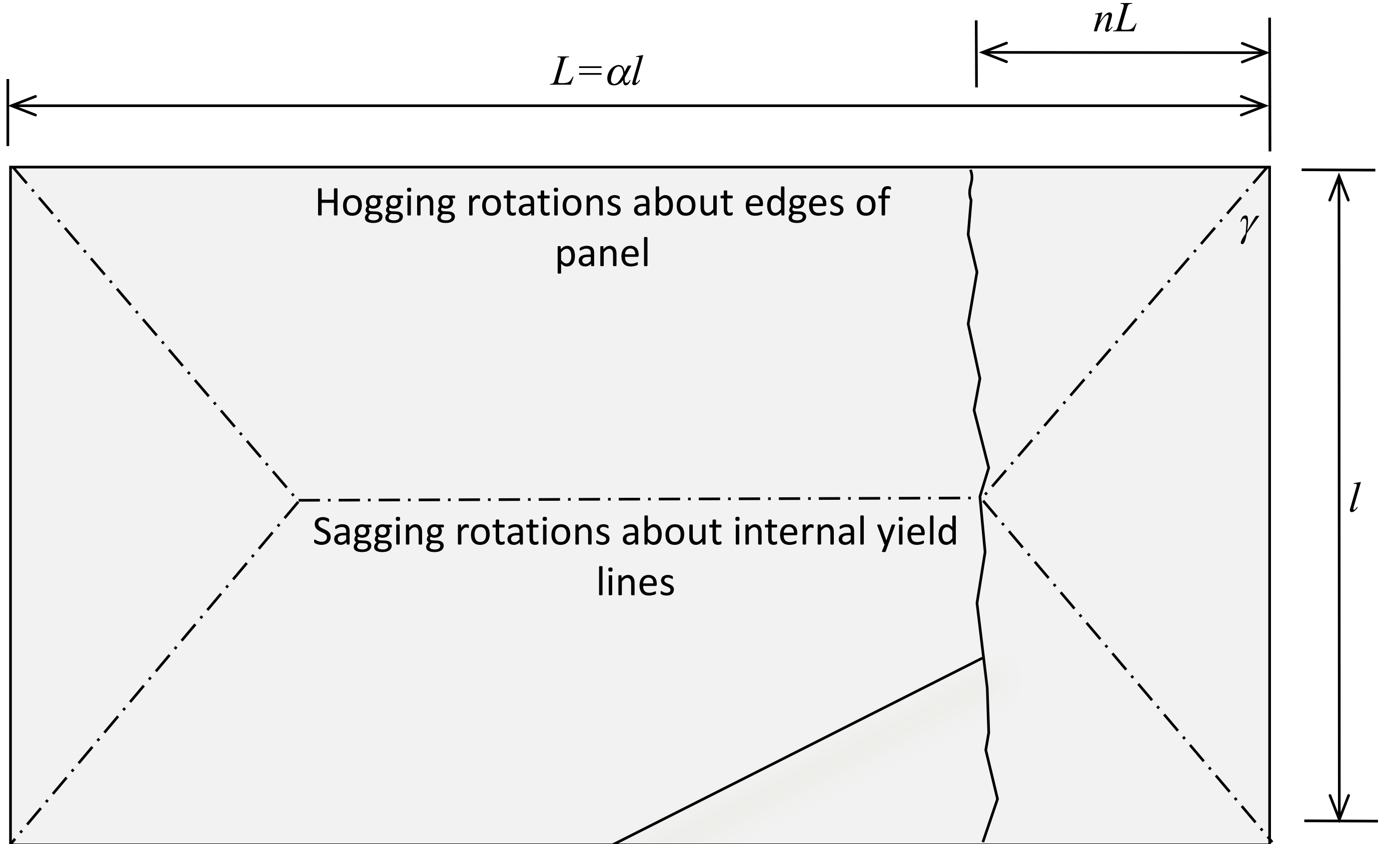
B. Hayes

Notation

Introduction

Existing methods of analysis

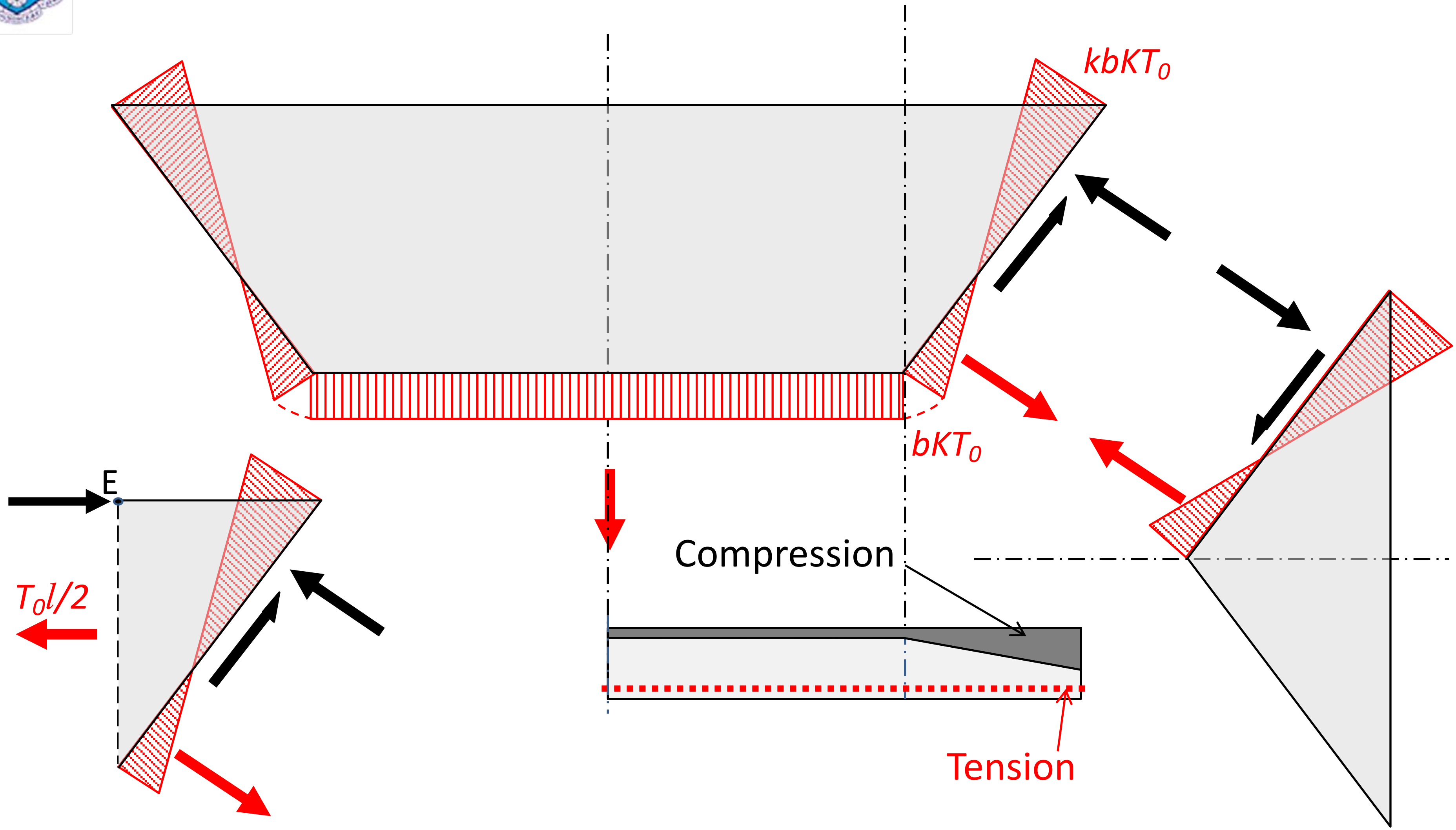
Small-deflection yield-line mechanism – slab only



Large-deflection failure crack sometimes observed in tests and used by Hayes.

Yield-line pattern is optimized for minimum concrete slab failure load.

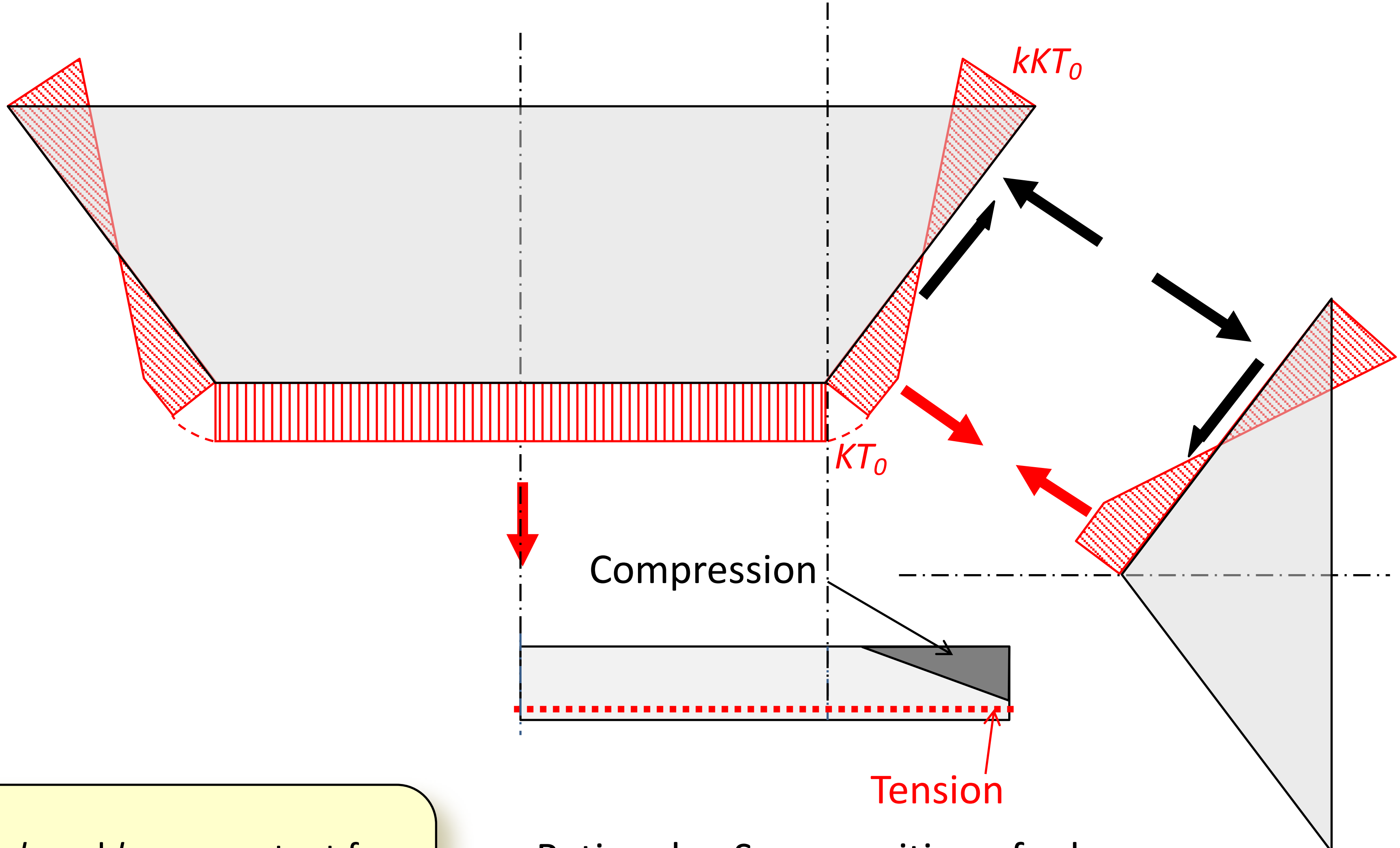
Equilibrium 1 – no through-depth YL cracks - Hayes



Criterion: Cracks from intersection. Moment equilibrium about E. Finds *b* and *k*.

Rationale: Superposition of rebar tension and concrete compression force/unit length.

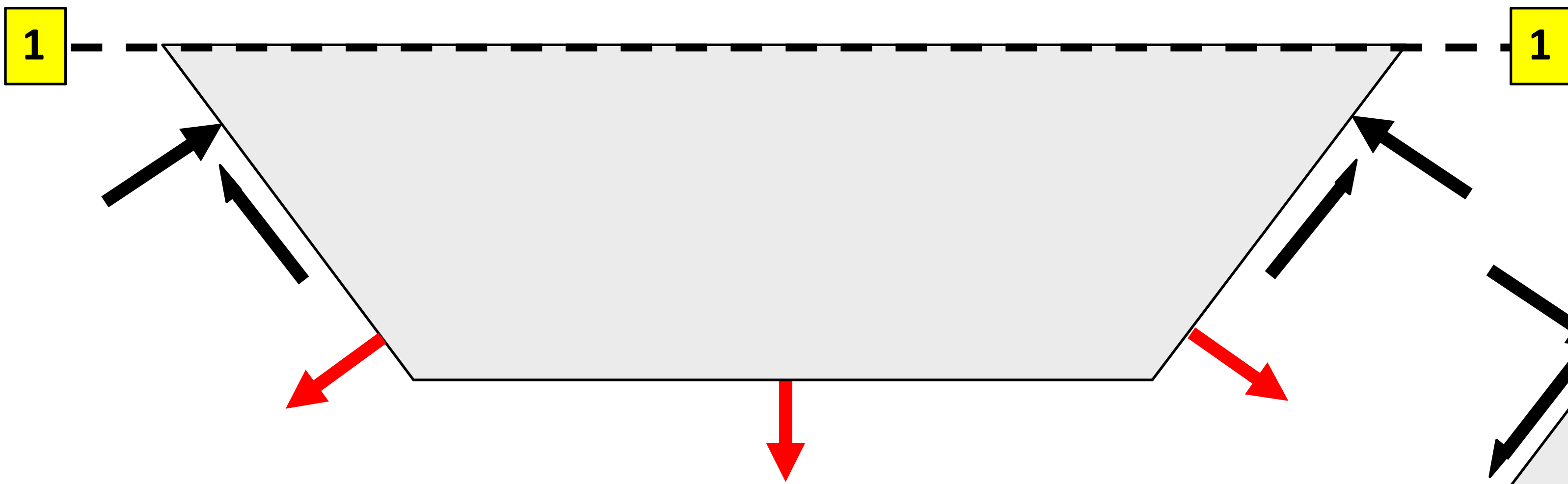
Equilibrium 2 – some through-depth YL cracks - Hayes

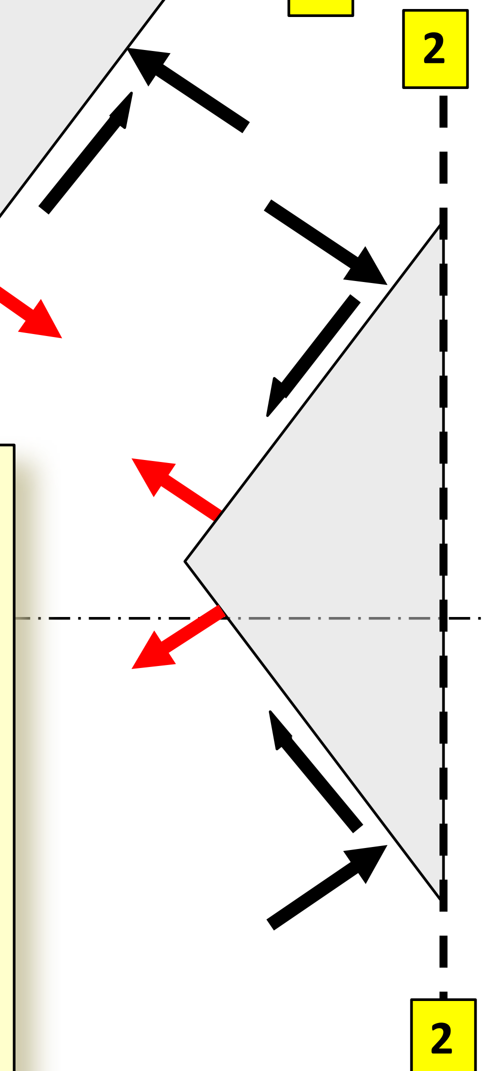


Both *b* and *k* are constant for each of the 2 cases. No variation with deflection.

Rationale: Superposition of rebar tension and concrete compression force/unit length.

Partial enhancement factors – both cases - Hayes





<u>“Membrane force” enhancements:</u>

e_{1m} Moment of membrane forces about 11/total resistance moment about x-axis at initial YL.

e_{2m} Moment of membrane forces about 22/total resistance moment about y-axis at initial YL.

These start at zero for zero deflection

<u>Resistance moment enhancement (reduction)</u>

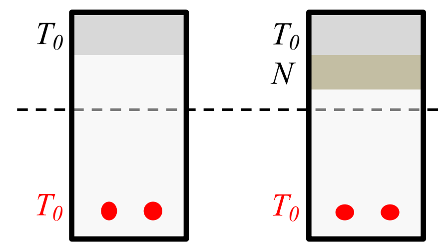
e_{1b} Proportional change of resistance moment about x-axis due to membrane compression.

e_{2b} Proportional change of resistance moment about y-axis due to membrane compression.

Bending “enhancements” - Hayes

Wood’s equation for reduction of moment capacity of a rectangular RC cross-section due to axial compression:

1. Long-span reinforcement: $$\frac{M}{M_0} = 1 + A \frac{N}{T_0} \quad B \frac{N}{T_0}^{\;2}$$



2. Short-span reinforcement: $$\frac{M}{M_0} = 1 + A' \frac{N}{KT_0} \quad B' \frac{N}{KT_0}^{\;2}$$

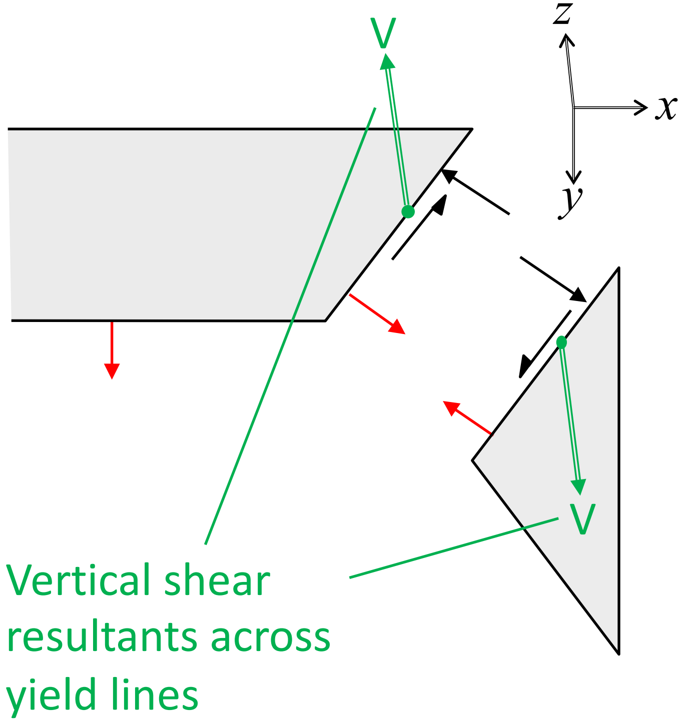
These are integrated in x- and y- directions respectively for the bending moments across the yield lines for Portions 1 and 2.

Forming an overall enhancement factor - Hayes

$$e_1 = e_{1m} + e_{1b}$$
$$e_2 = e_{2m} + e_{2b}$$

These are nearly always unequal (**WHY?**). Put together as

Overall enhancement factor $$e = e_1 - \frac{e_1 - e_2}{1 + 2\mu a^2}$$



These don't include any vertical shear between the facets. If these are included there is only one enhancement factor.
(Tony Gillies 2015)

New enhancement Factor equivalent to $$e = e_1 - \frac{e_1 - e_2}{1 + 2\mu n a^2}$$

Any problems so far?

- The membrane traction distribution is an assumption.

  It corresponds to unfractured mesh and either:

  - No through-depth cracks along yield lines.
  - Partial through-depth cracks along yield lines.

- Both of these distributions apply only to the case where a lateral through-depth crack has formed across the short span through the YL intersection.

- Distribution is fixed for each case. Enhancement factor starts below 1.0 – actually at zero.

- Internal forces don't depend on deflection.

Structural fire resistance methods for composite floors

BRE Method (Bailey 2000)

- Amended version of Hayes's method.
- *Fire Safe Design (SCI P288)* checked using BRE-Bailey design method.

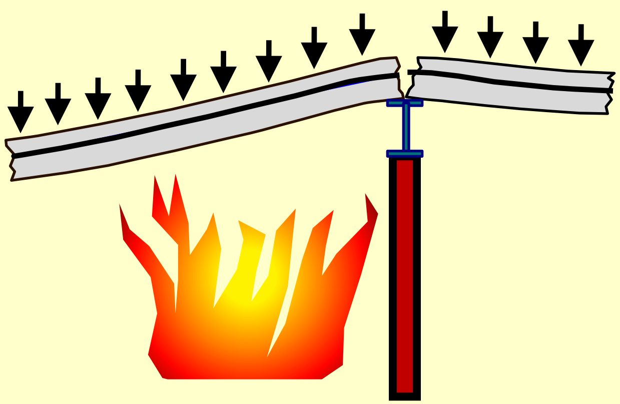
New Zealand SPM (Clifton 2006)

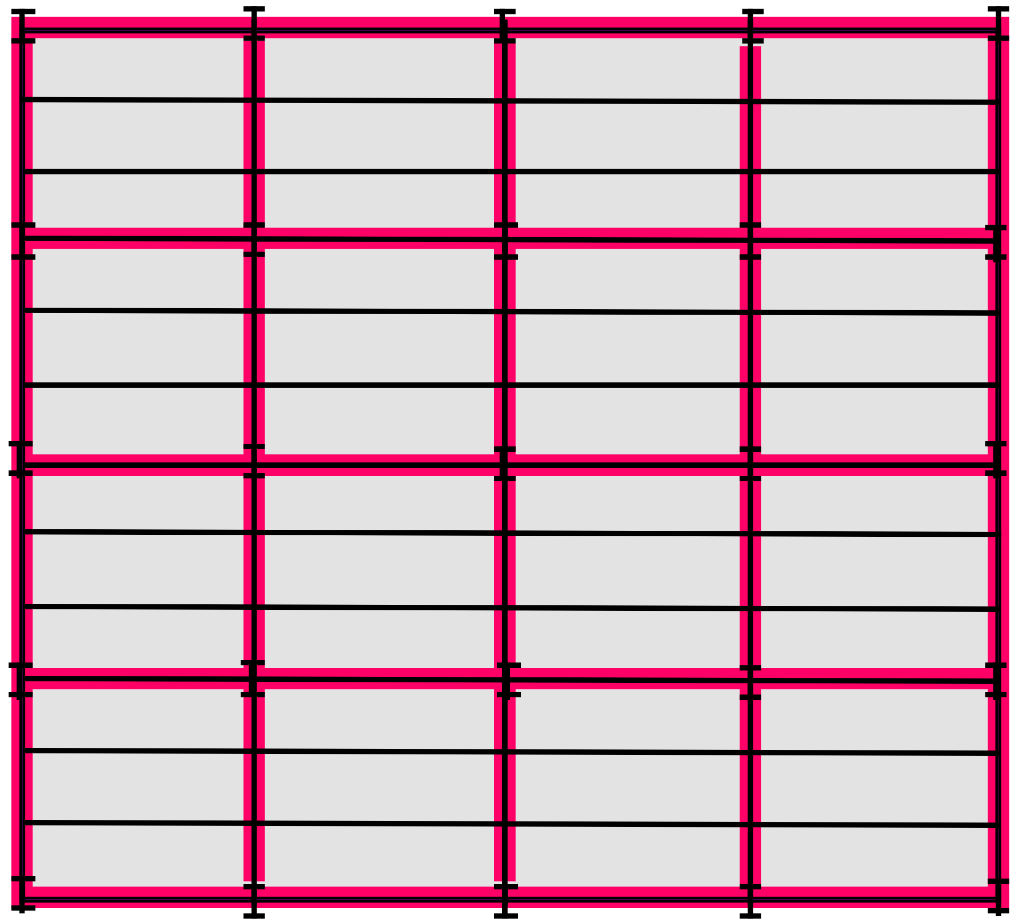
FRACOF (2011)

- Based on a European project.
- Almost identical to BRE method. A few changes to safety factors, extra deflection check.

Typical design strategy for TMA

- Protect members on column gridlines.
- Leave intermediate secondary beams unprotected.
- Design individual panels without continuity.



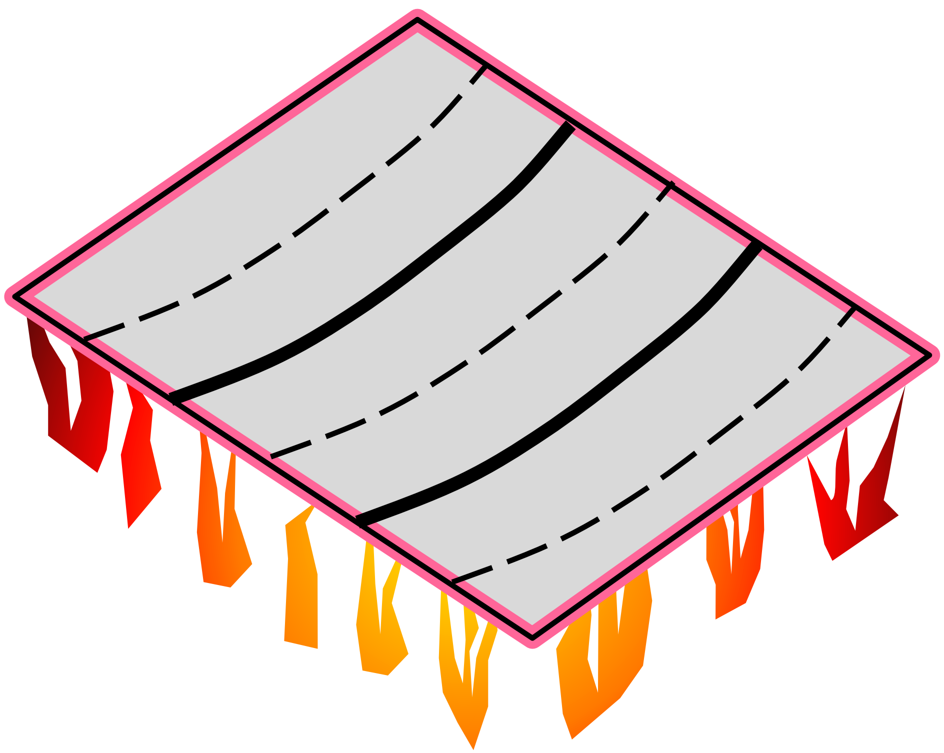


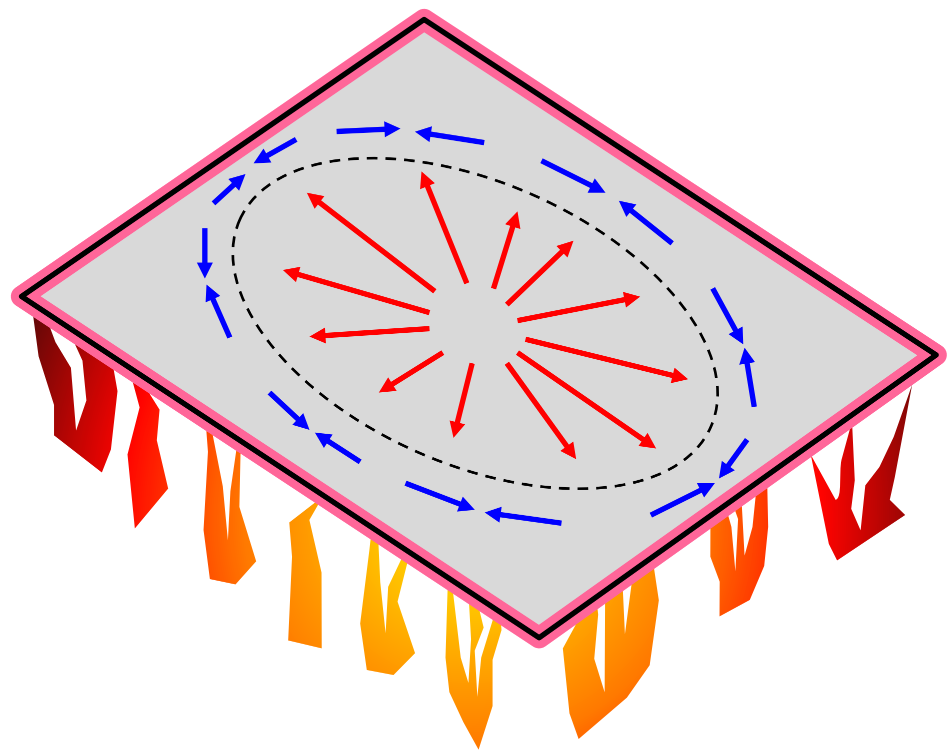
BRE/FRACOF method

- Unprotected composite beams at high temperature carry some of the load as simply supported.

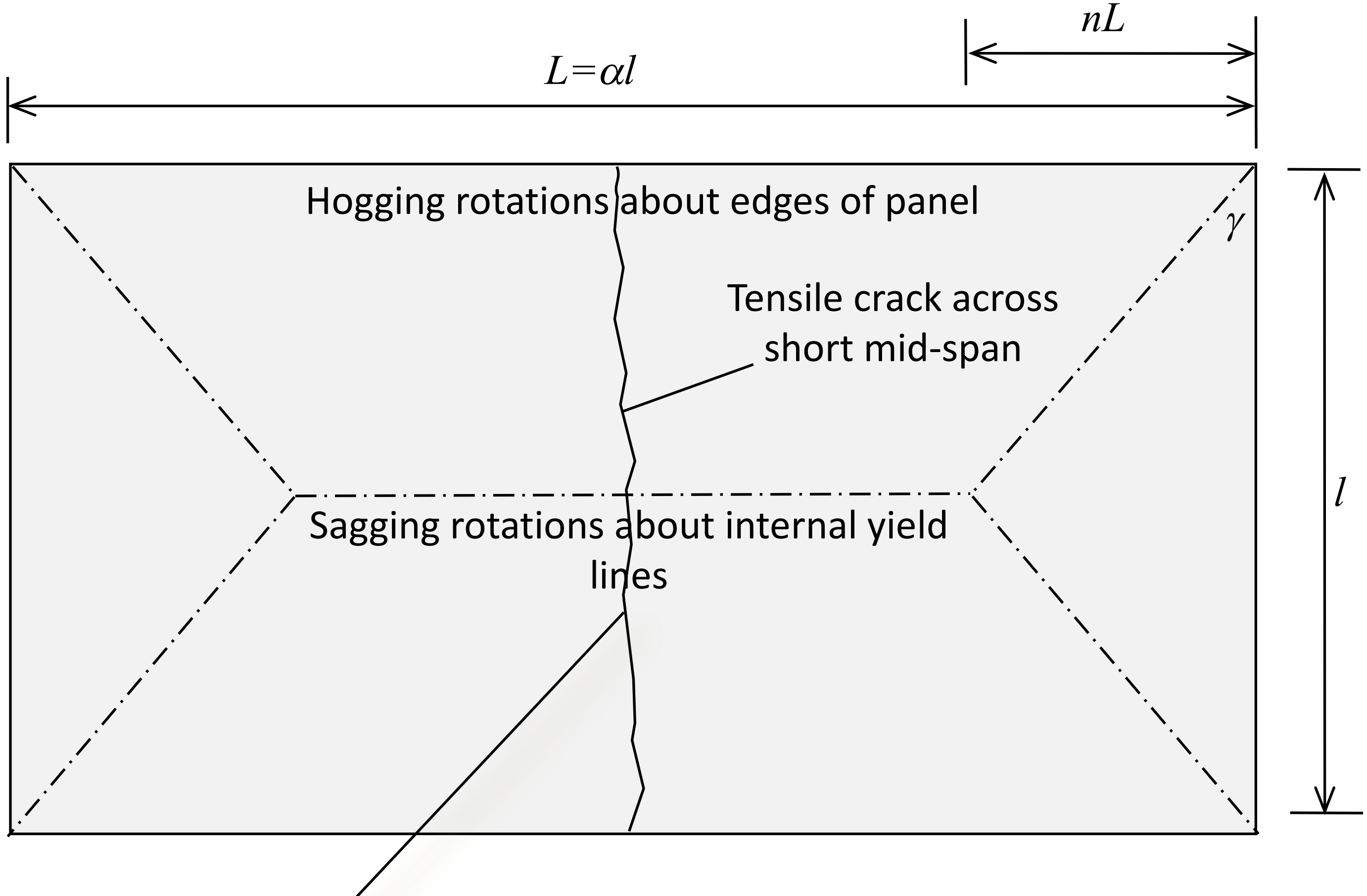
+

- Concrete slab carries remaining load in tensile membrane action. Needs enough deflection.





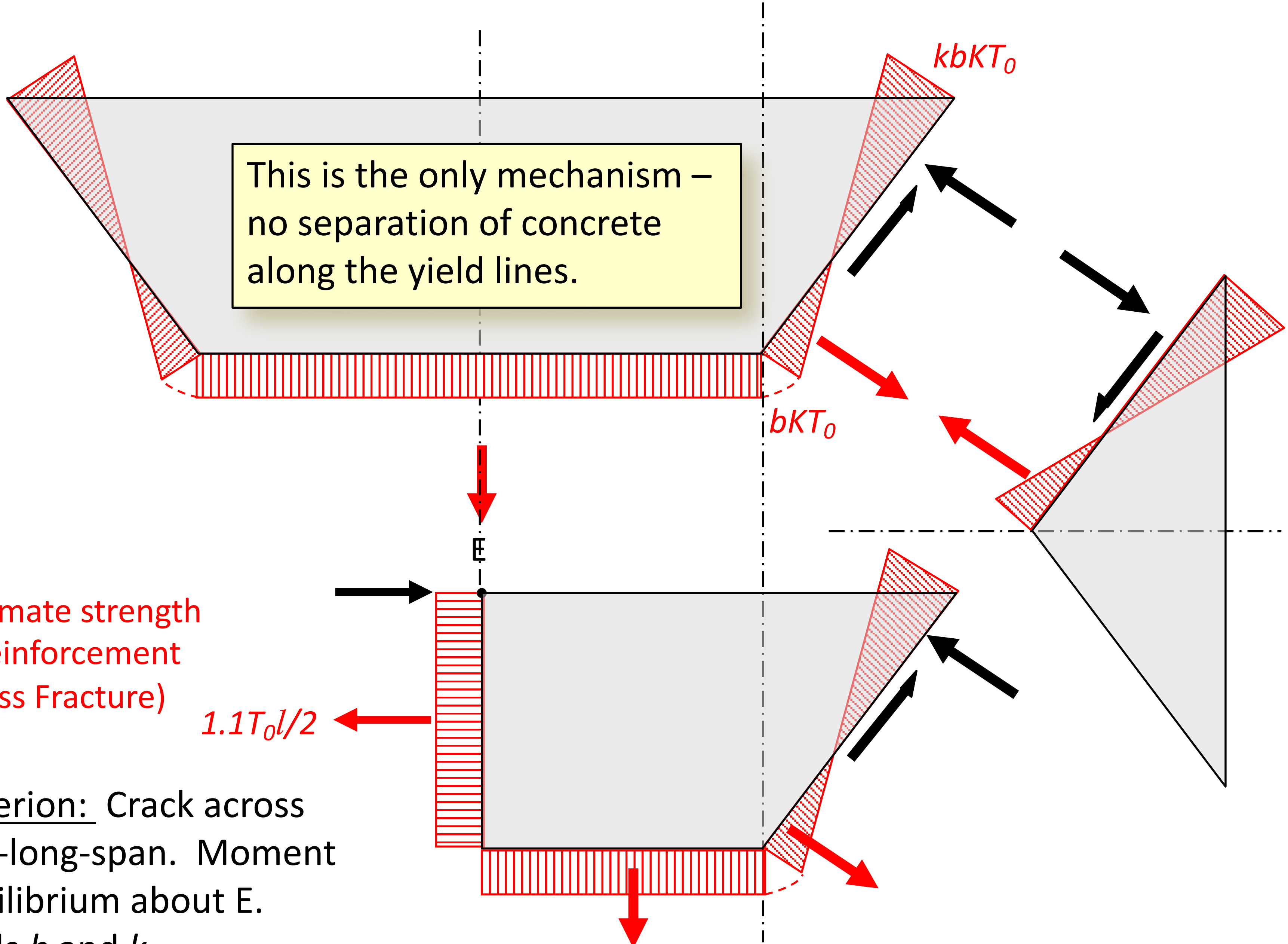
Small-deflection yield-line mechanism – slab only BRE/FRACOF



Large-deflection failure crack observed in tests and used in Bailey/BRE, FRACOF and NZ SPM.

The analysis is based on the optimal yield-line pattern for the concrete slab without considering the steel beams.

Force equilibrium – no through-depth YL cracks – BRE etc



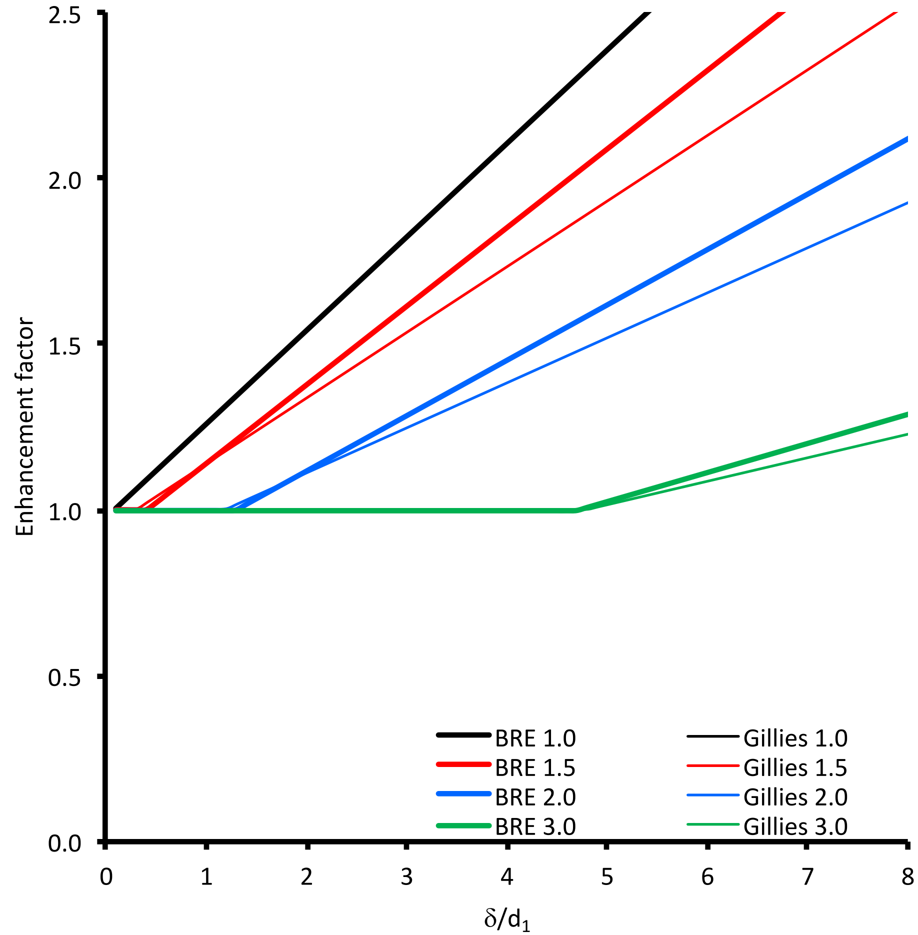
(Ultimate strength of reinforcement across Fracture)

<u>Criterion:</u> Crack across mid-long-span. Moment equilibrium about E. Finds *b* and *k*.

TMA enhancement calculations – BRE/FRACOF

Similarly to Hayes:

- Horizontal force equilibrium assuming mid-span crack. (But only the linear membrane traction distribution).
- Separate "membrane" enhancements e_{1m} and e_{2m} by moments about long and short edges.
- Add "bending" enhancements e_{1b} and e_{2b} to make e_1 and e_2.
- Overall enhancement factor $e = e_1 - \frac{e_1 - e_2}{1+2\mu a^2}$
- … or Gillies $e = e_1 - \frac{e_1 - e_2}{1+2\mu n a^2}$
- Cutoff at enhancement 1.0 for aspect ratios > 1.0.

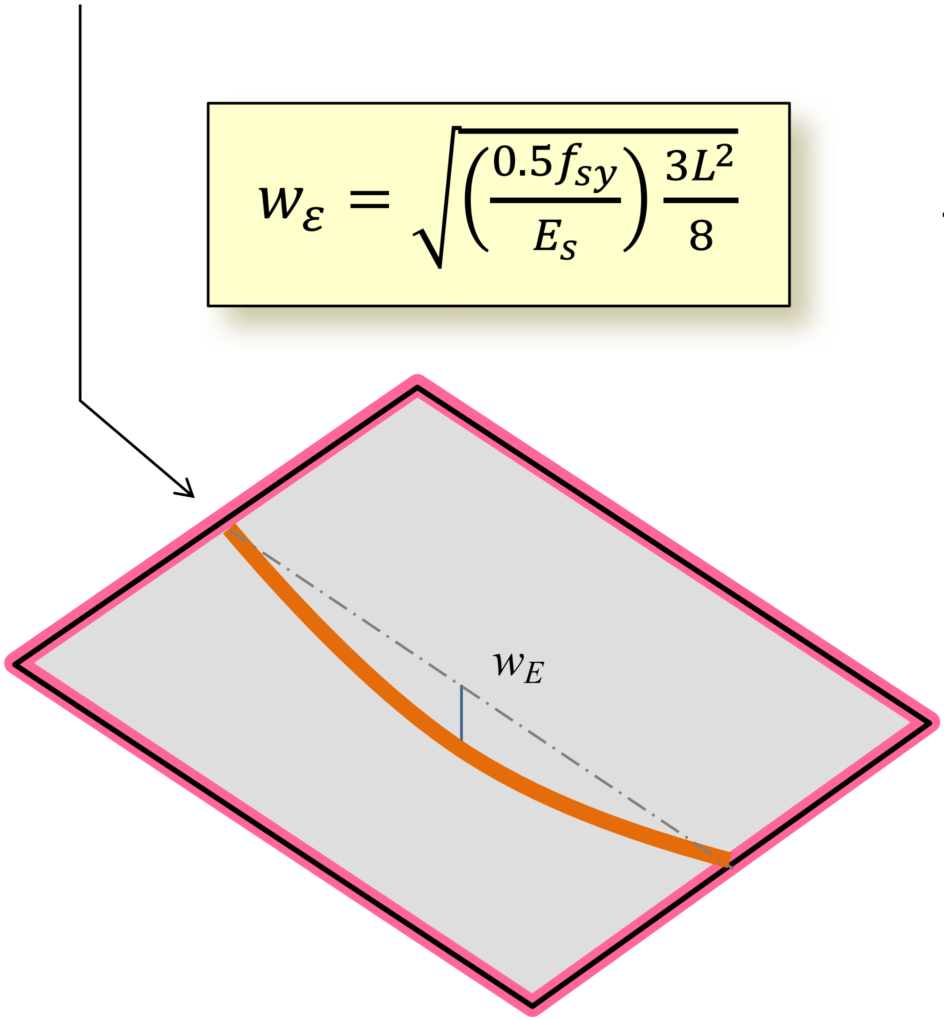


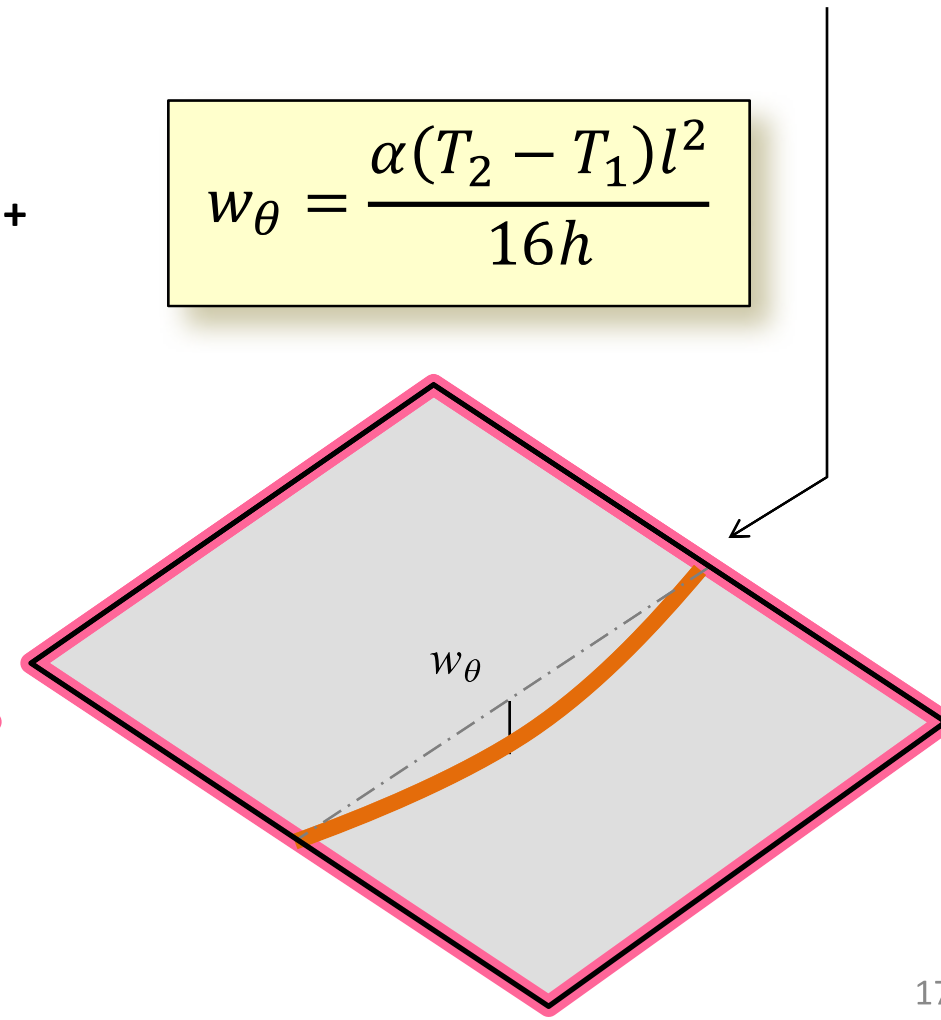
Limiting deflection (central cracking) criterion

$$w_{\varepsilon} = \sqrt{\left(\frac{0.5 f_{sy}}{E_s}\right)\frac{3L^2}{8}}$$

+

$$w_{\theta} = \frac{\alpha(T_2 - T_1)l^2}{16h}$$





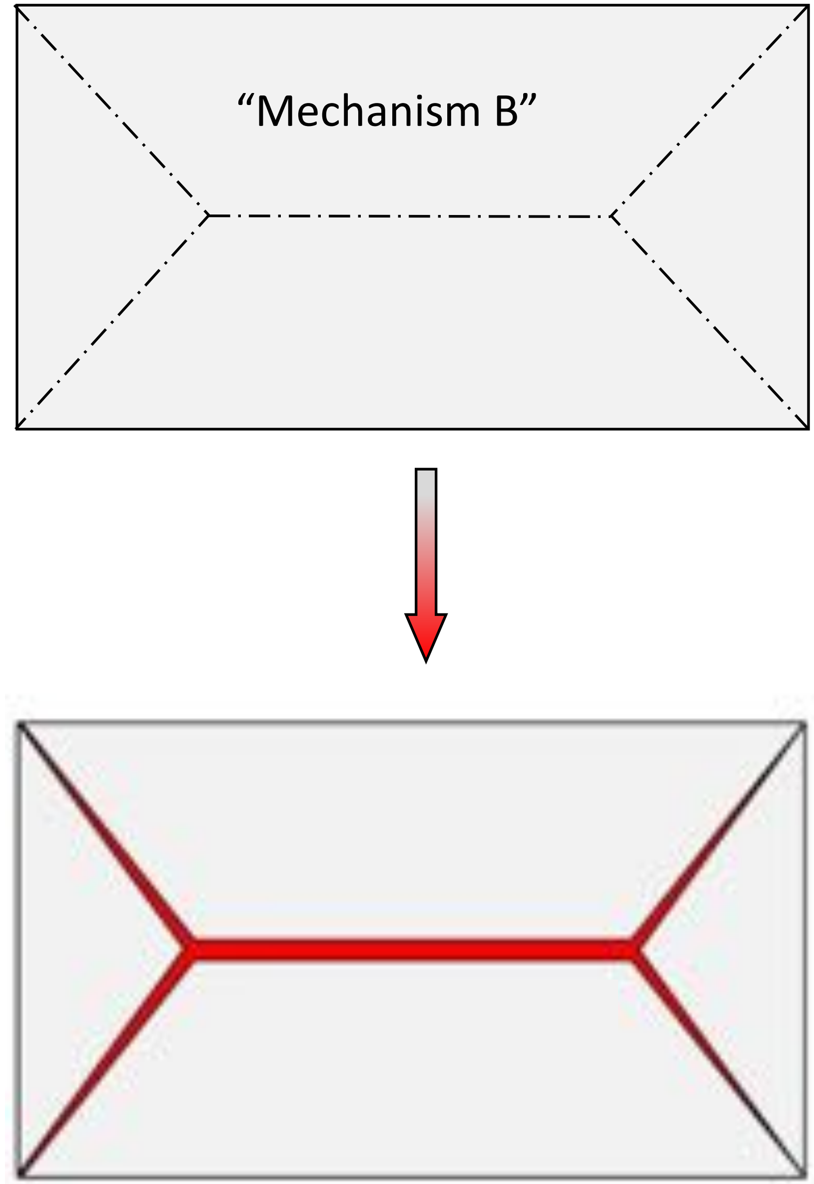
Back to basics

Increasing deflection of yield-line mechanism

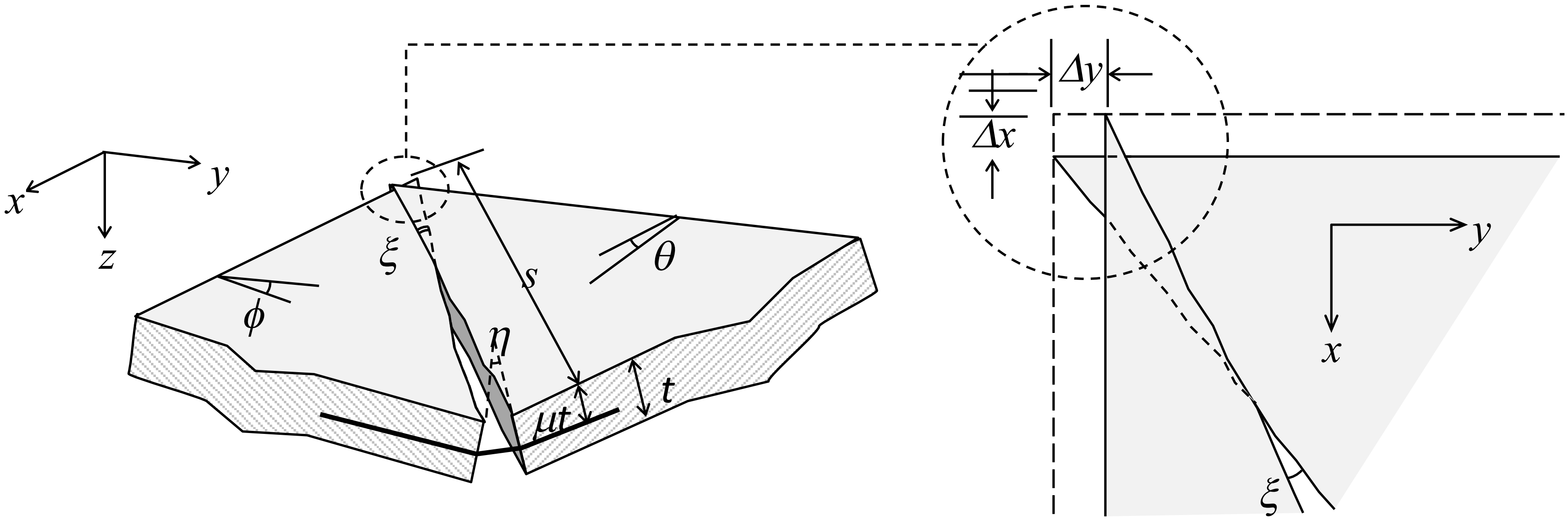
Yield-line mechanism is a plastic bending mechanism at small deflections. Yield lines are essentially discrete cracks.

As deflections start to increase the yield-line pattern increases the rotations of its flat facets, with the rebar yielding until it fractures.

So the initial large-deflection mechanism is this one.



Geometry of yield-line crack opening

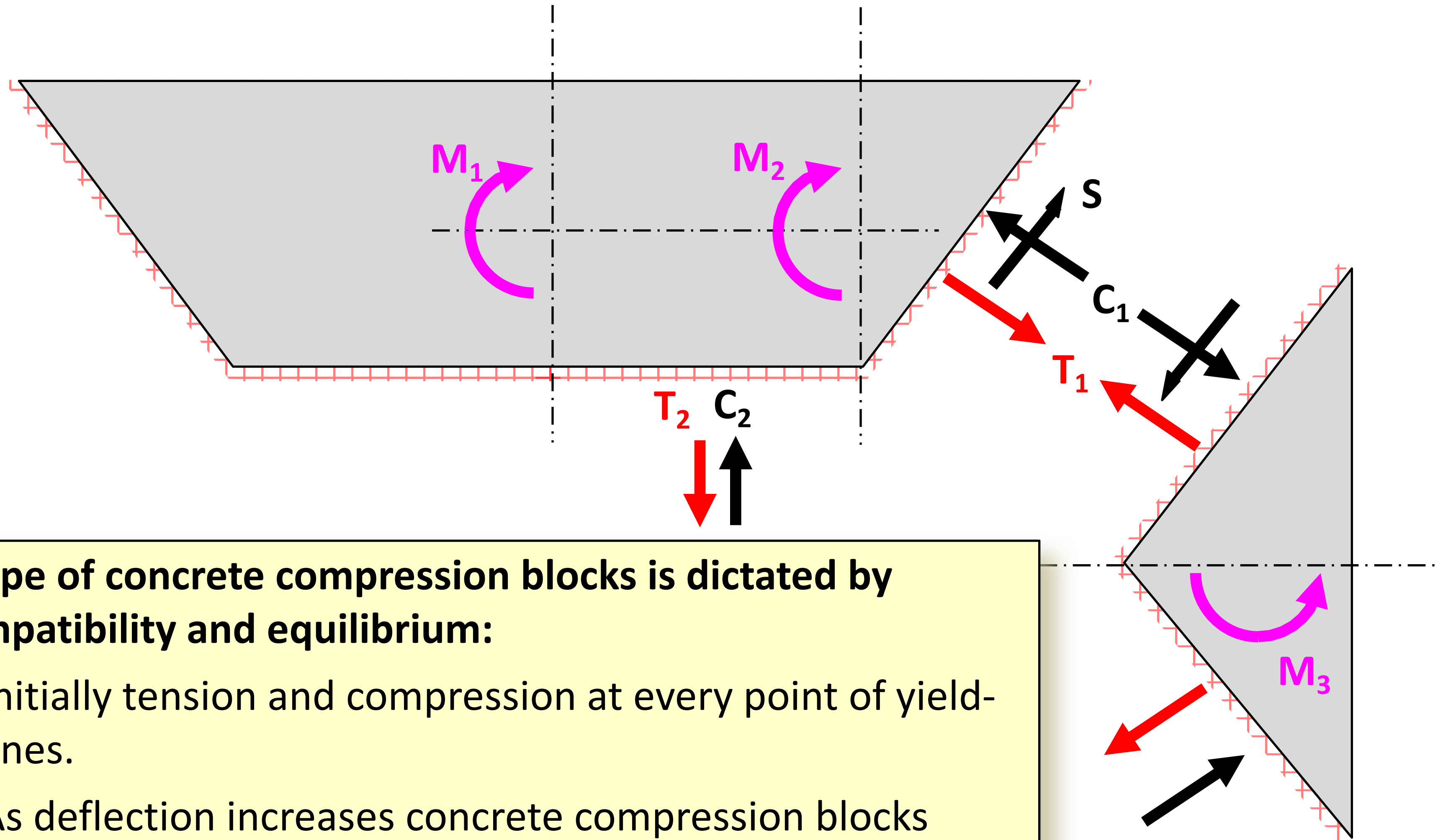


CRACK OPENING AT REBAR LEVEL

TOP SURFACE OF SLAB

As deflections start to increase the yield-line pattern increases the rotations of its flat facets, and rebar yields across cracks until it fractures.

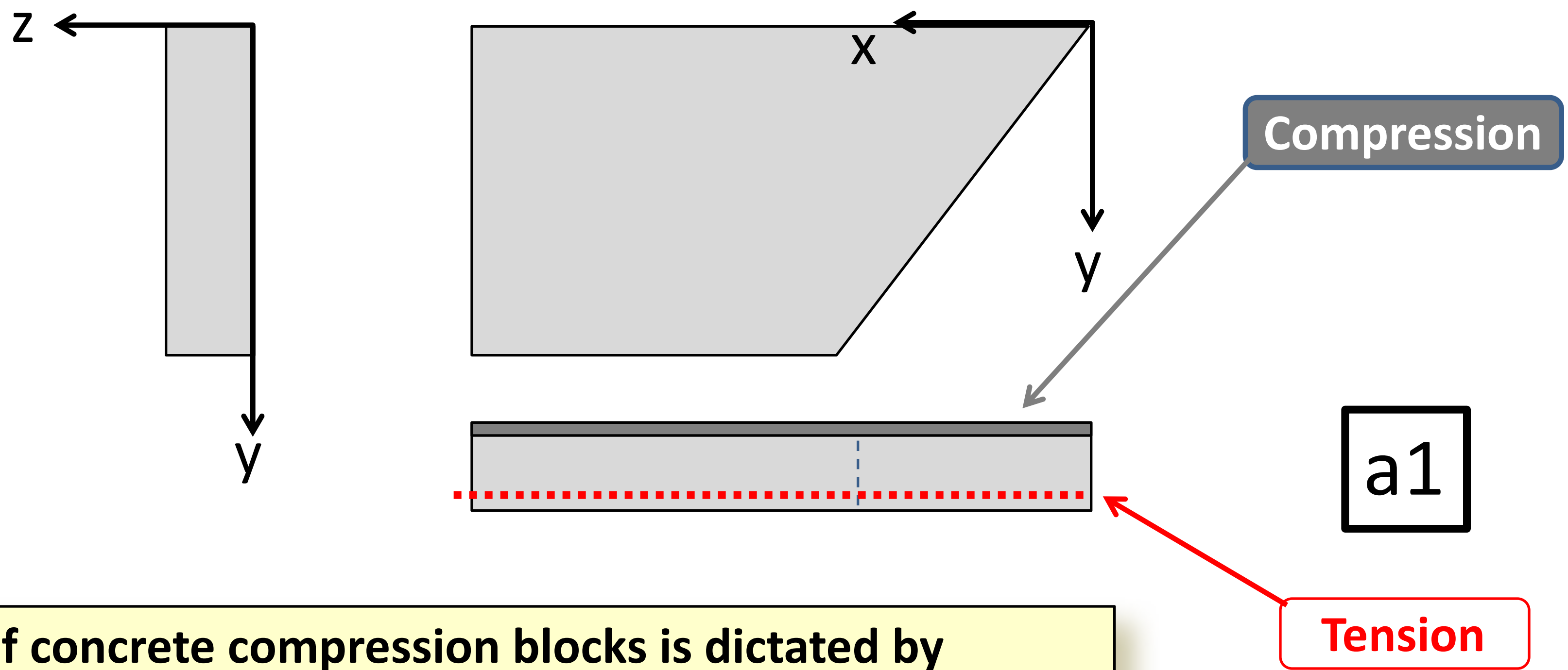
Force equilibrium of Mechanism B



Shape of concrete compression blocks is dictated by compatibility and equilibrium:

- Initially tension and compression at every point of yield-lines.
- As deflection increases concrete compression blocks concentrate towards slab corners, rebar fractures when its strain exceeds its ductility.
- No tension within compression blocks

Change of stress blocks – ductile y-reinforcement



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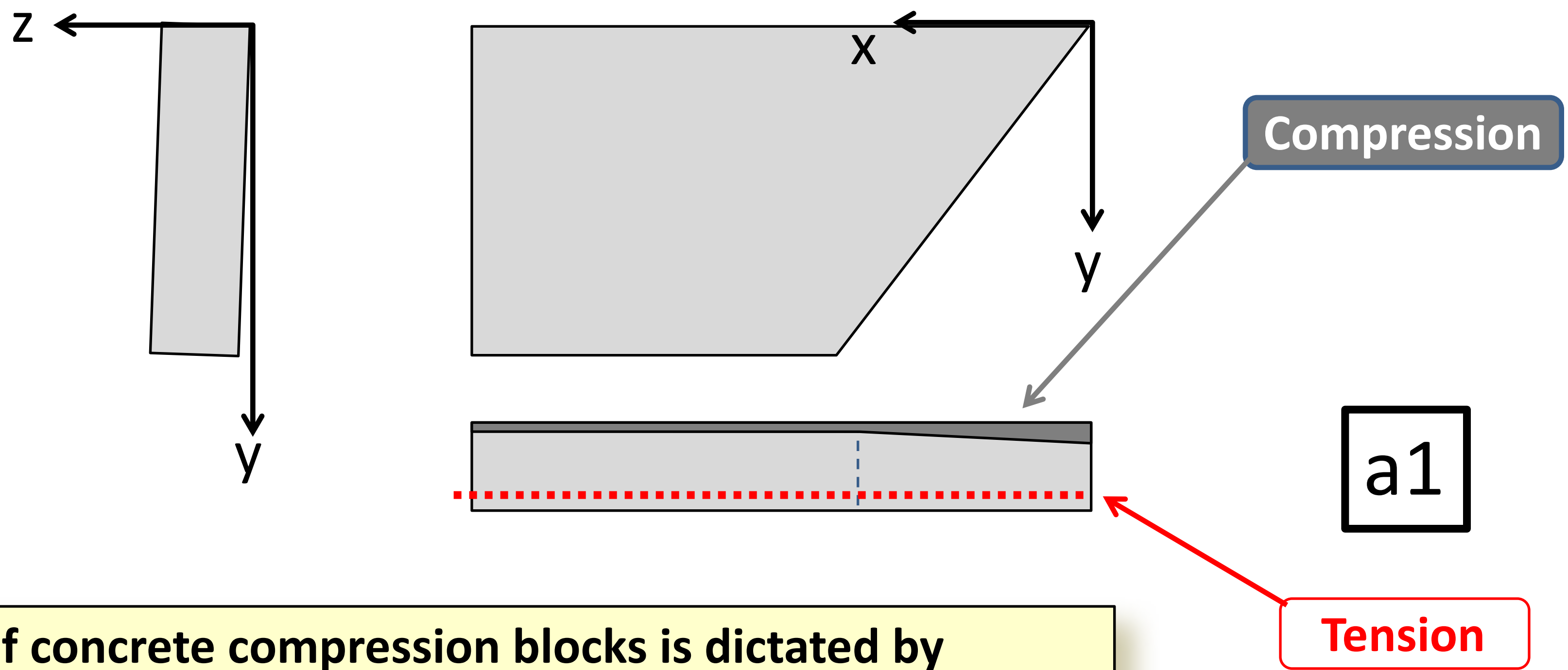

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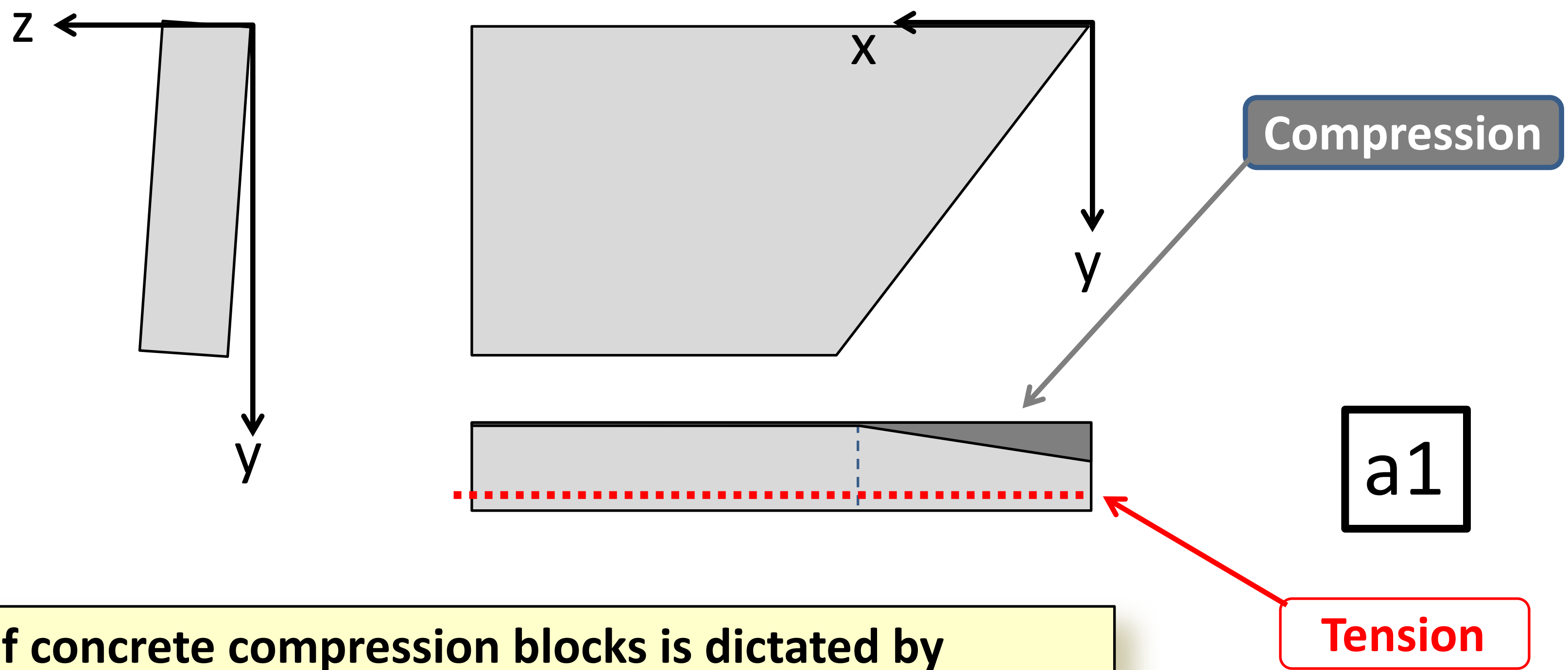

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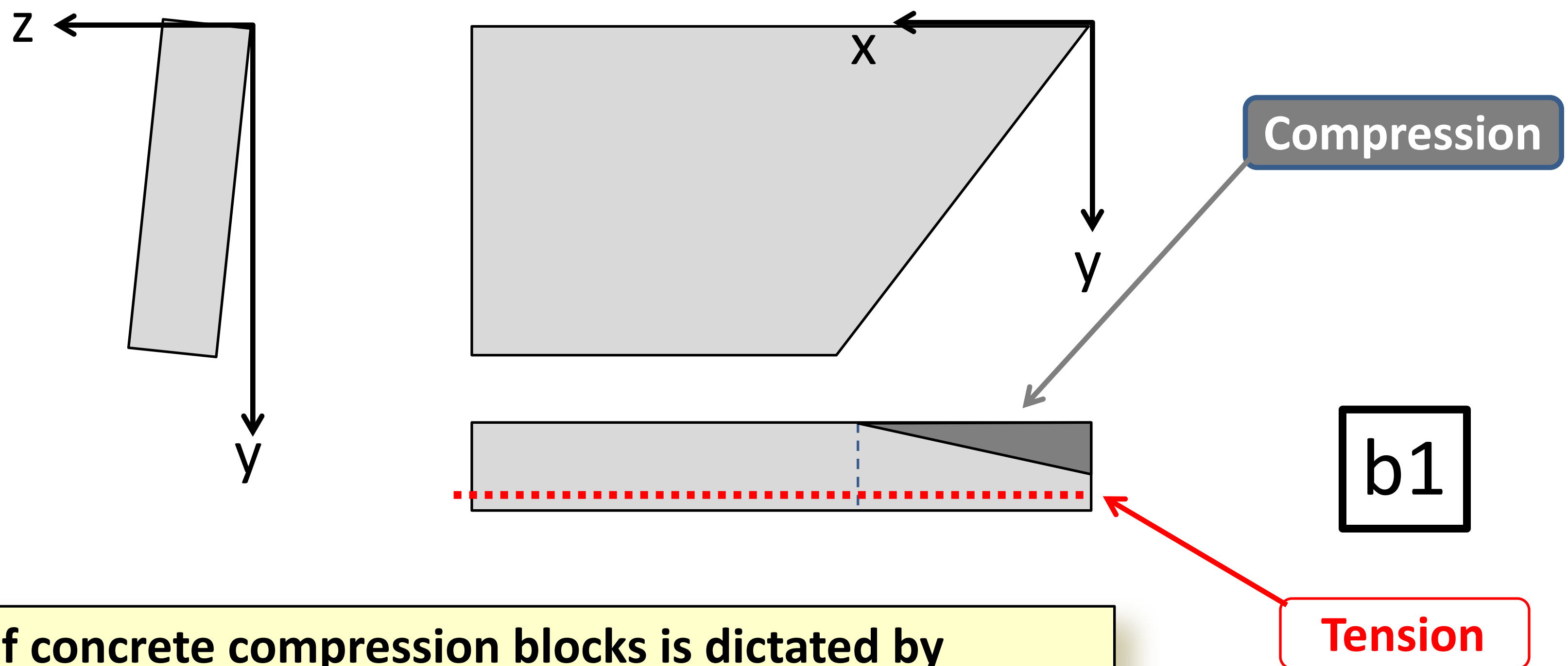

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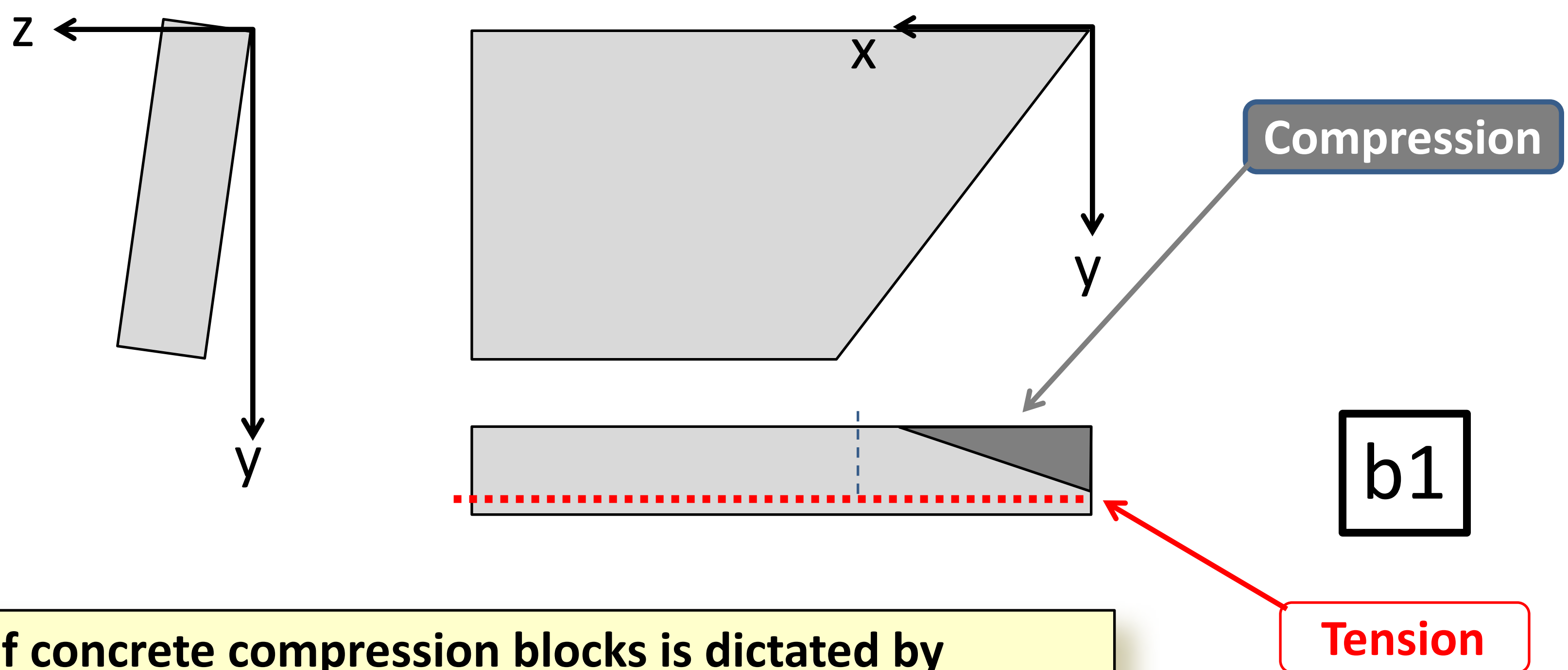

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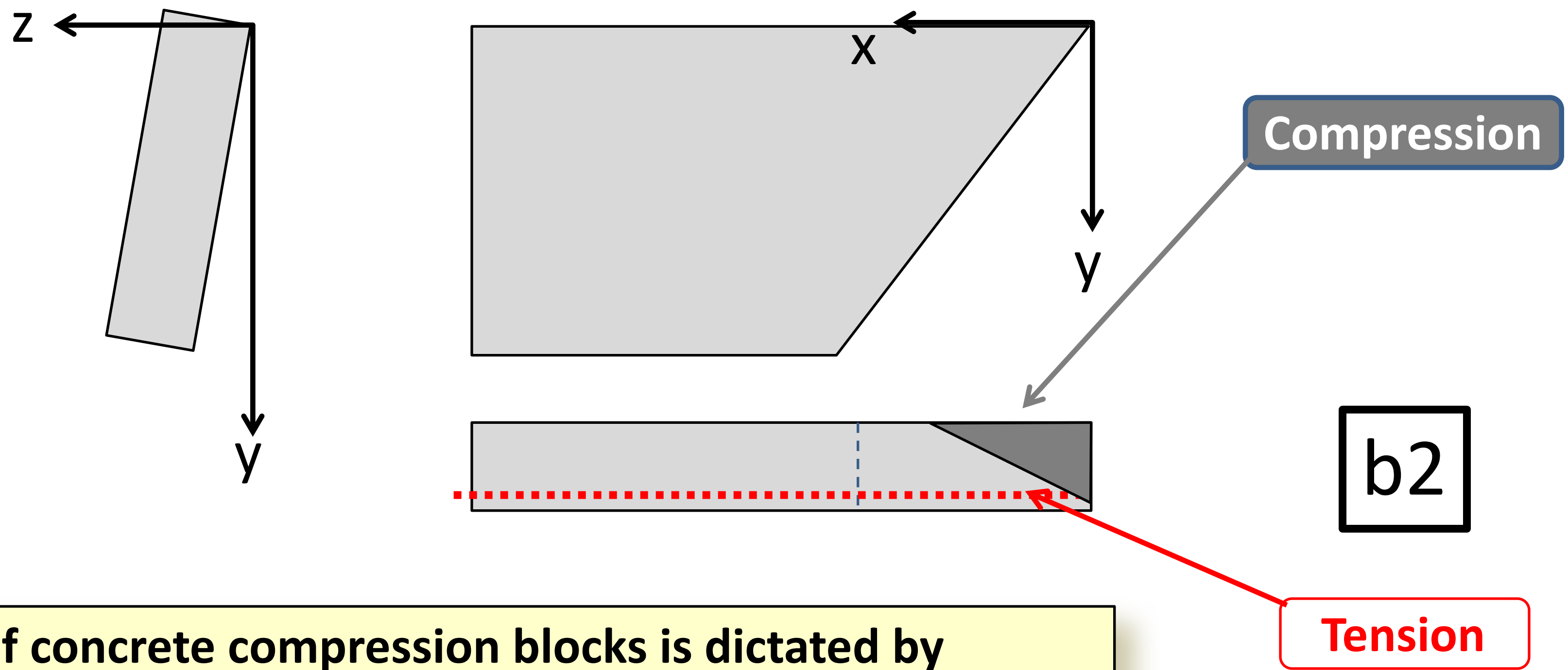

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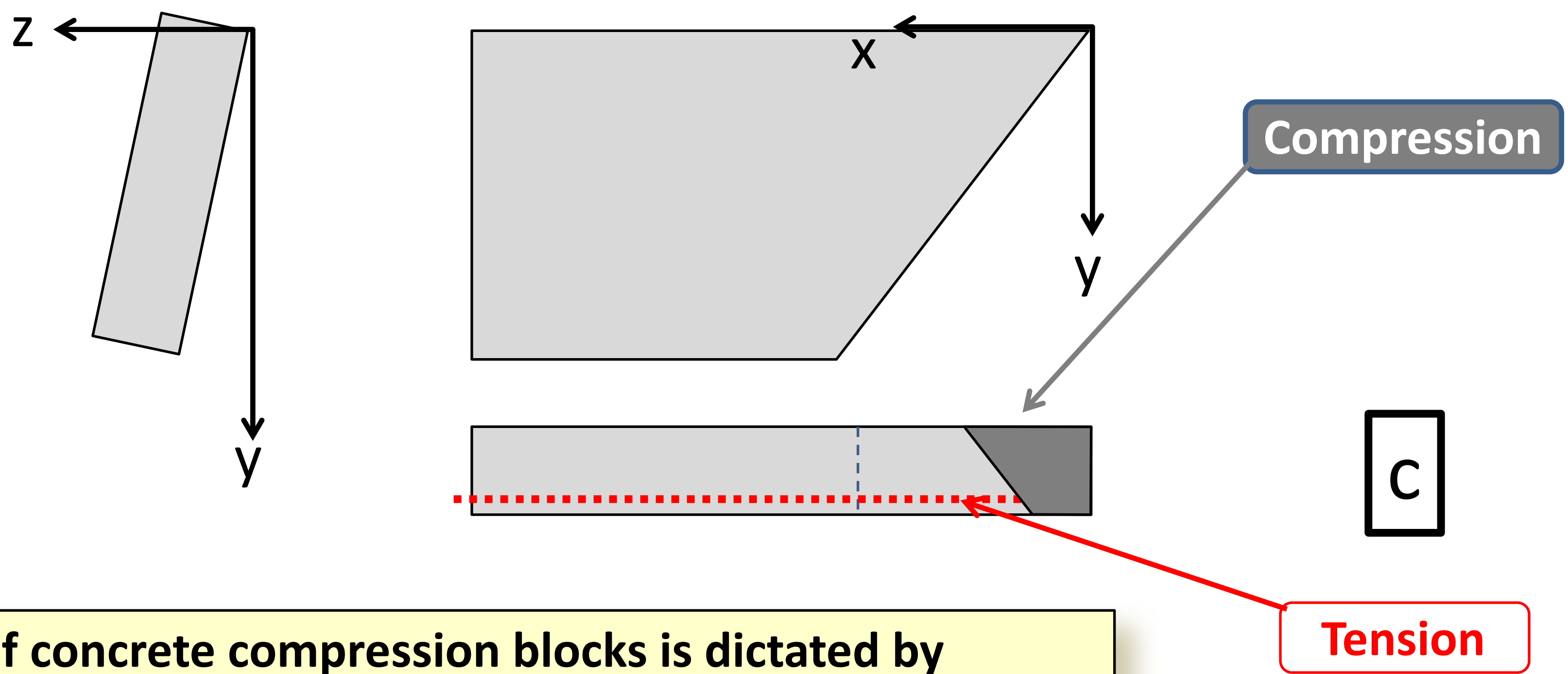

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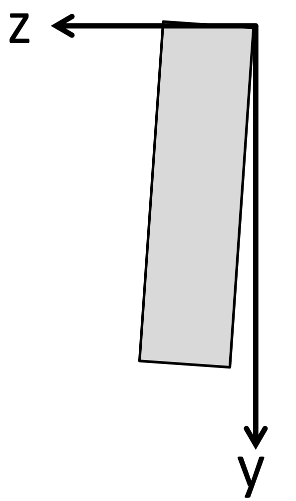
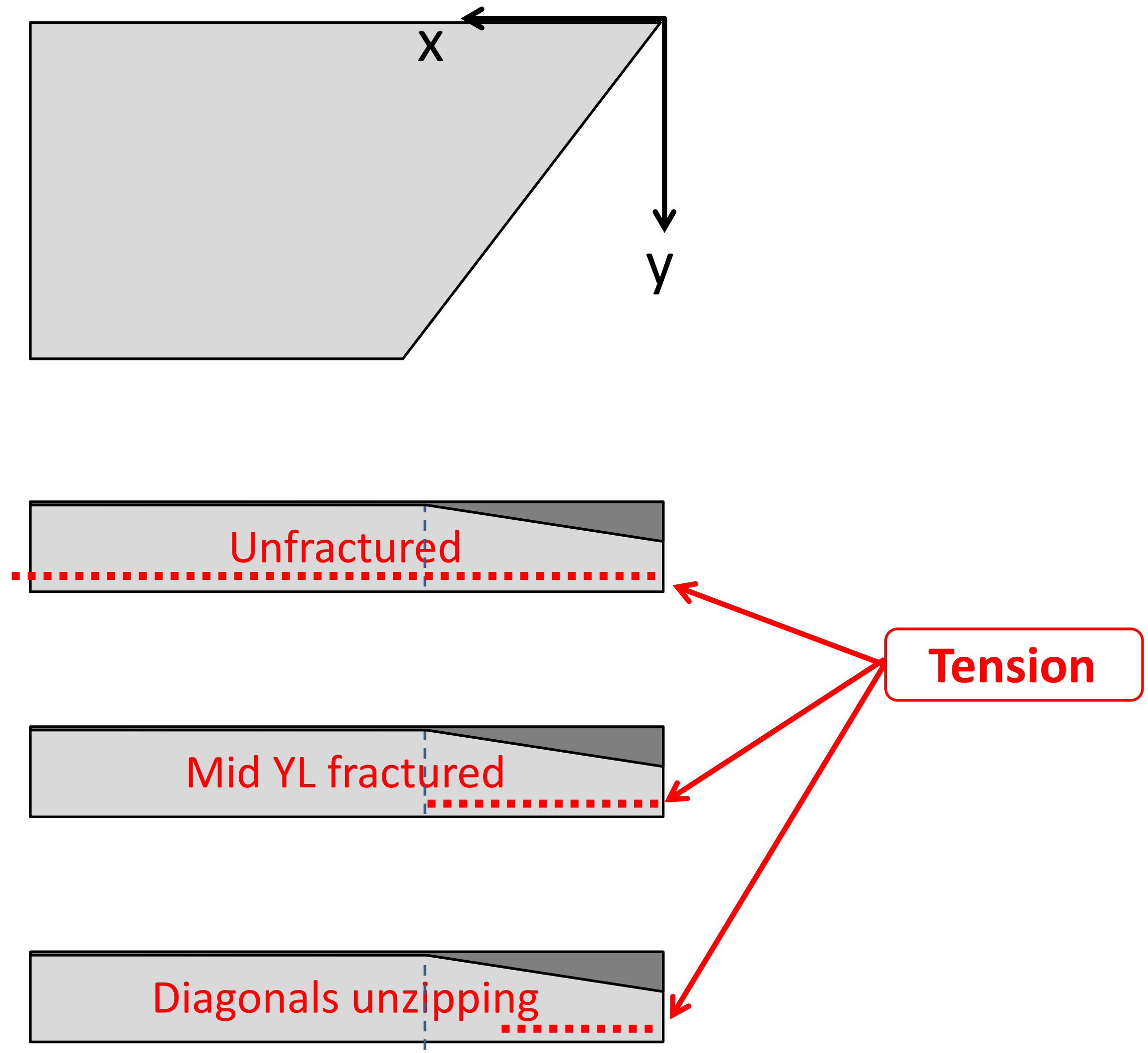
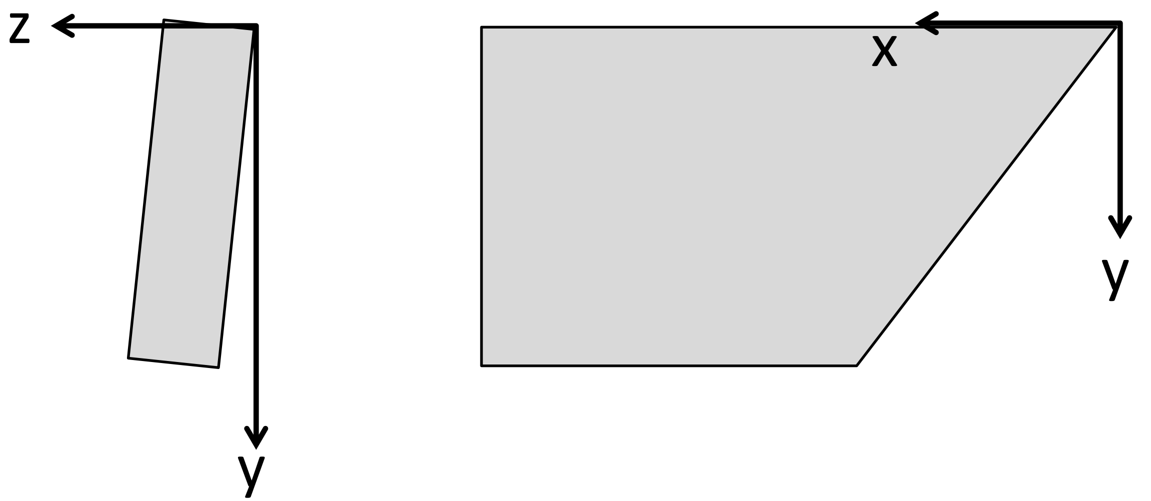
Change of stress blocks – possibilities with less ductility

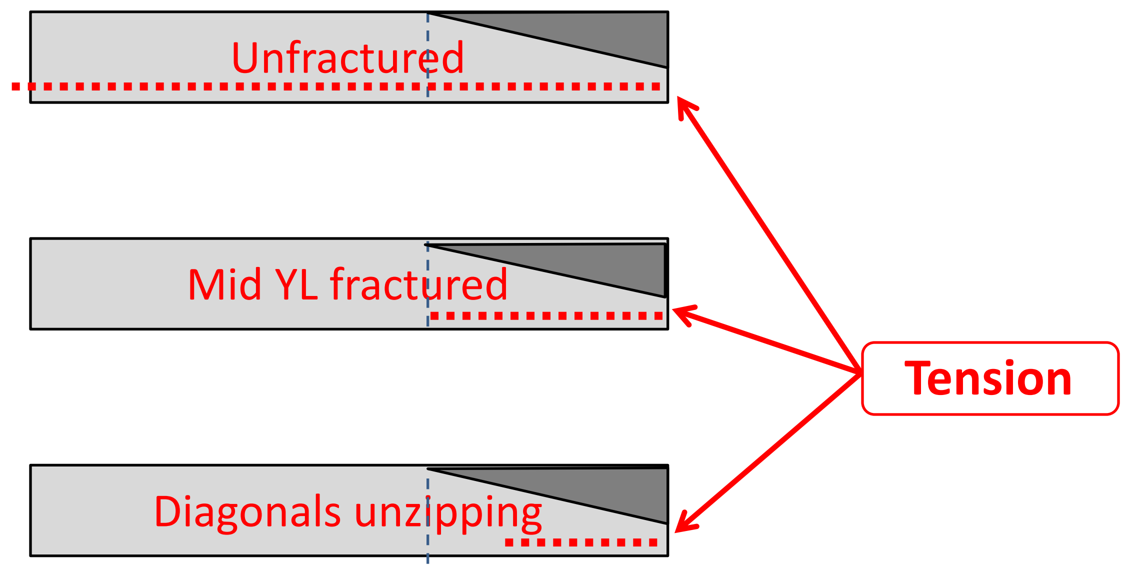




With different rebar ductility, mesh can either fracture abruptly or progressively at any stage.

Change of stress blocks – possibilities with less ductility





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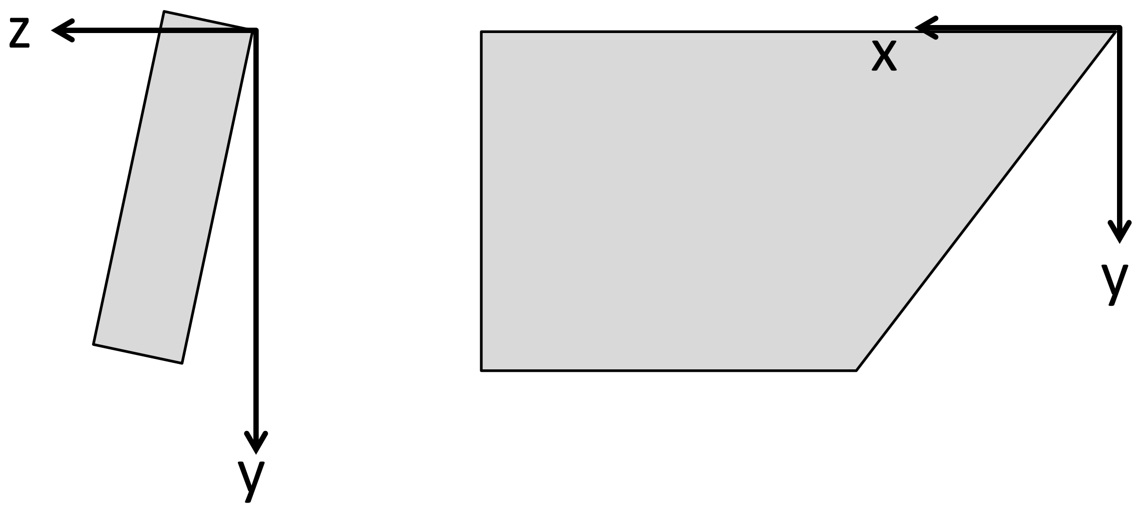

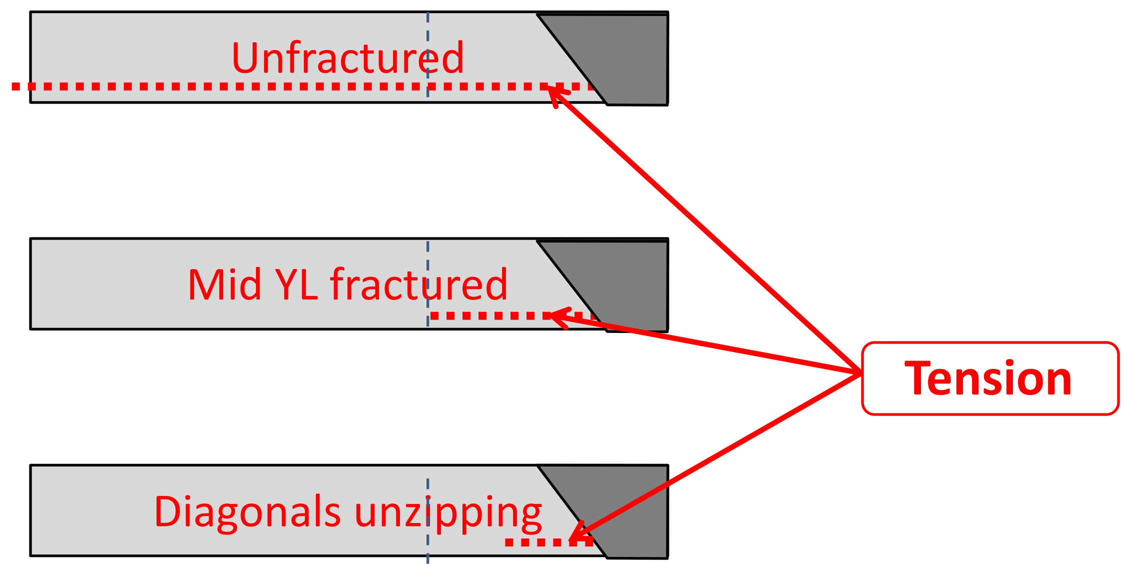

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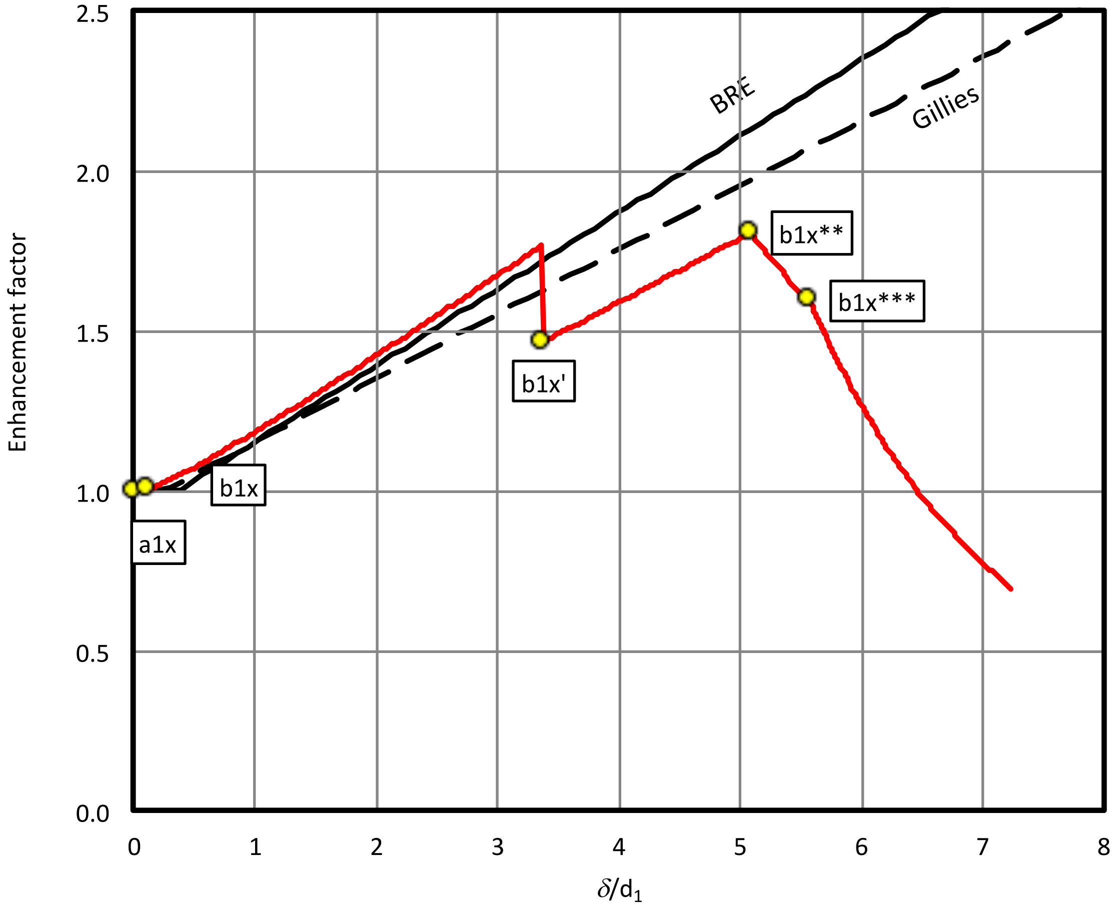
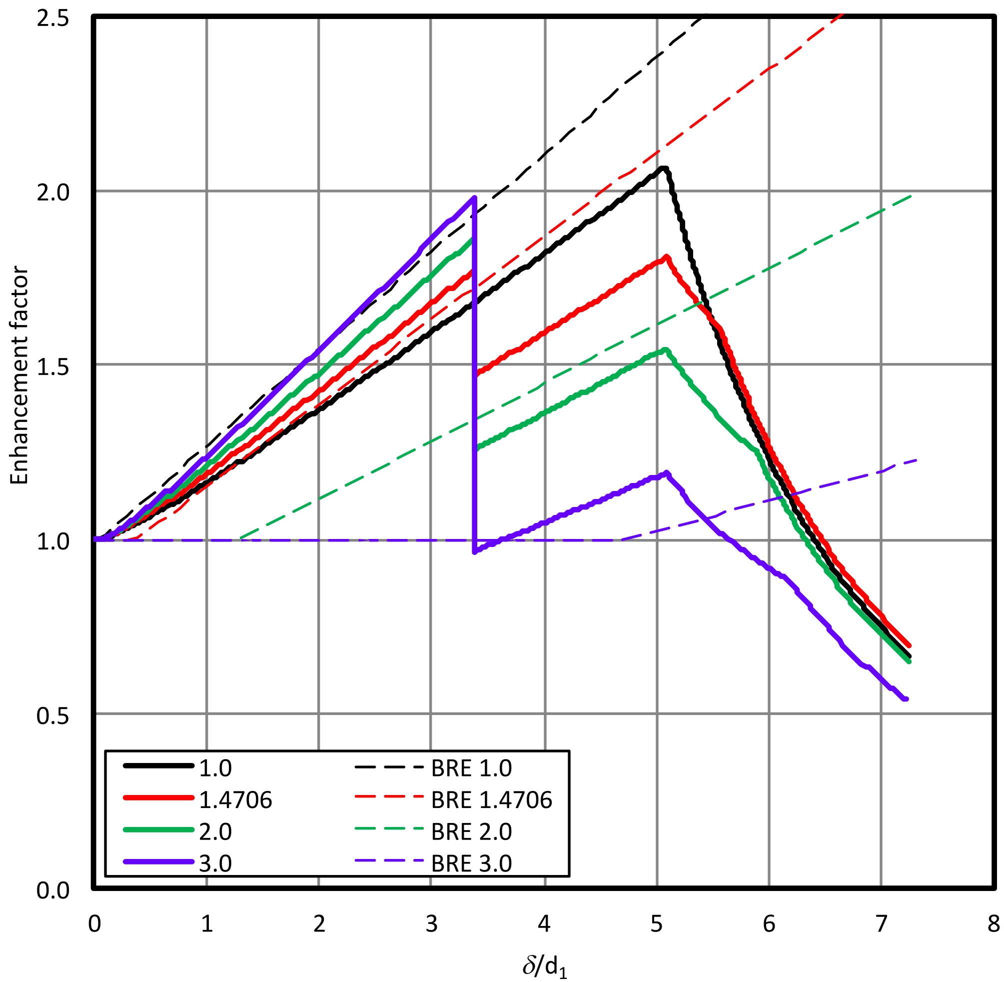
Garston test comparison

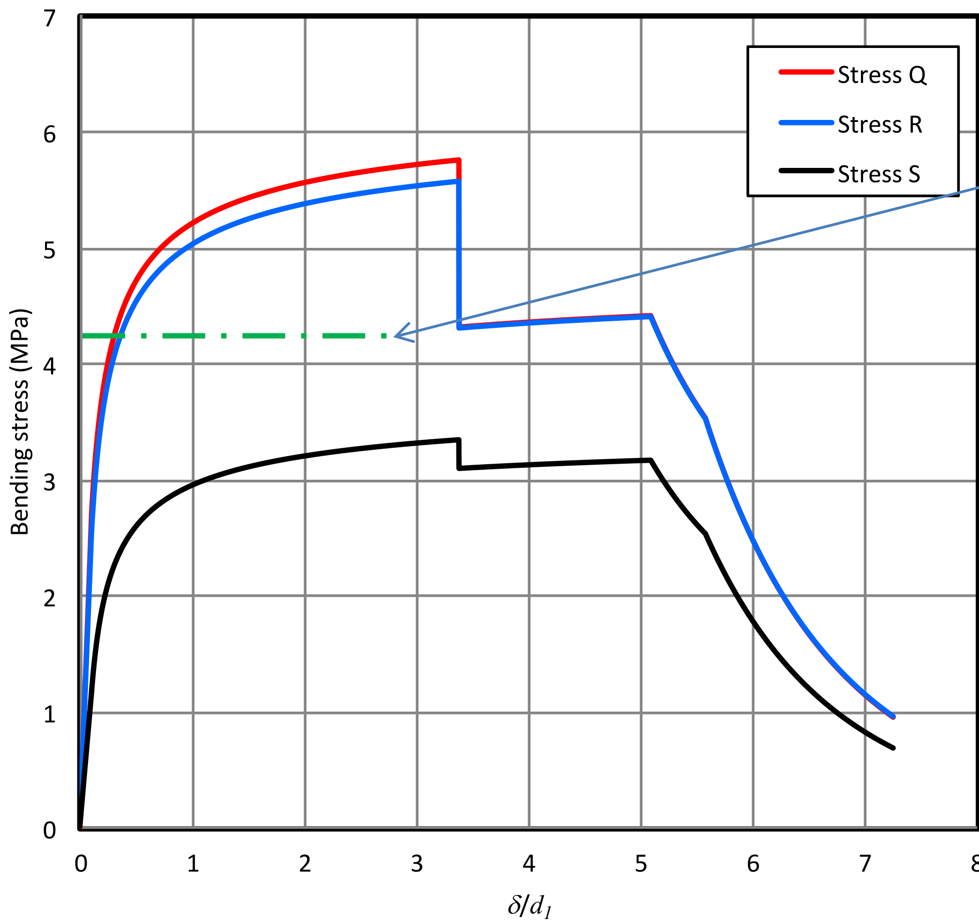


- Slab aspect ratio 1.4706 (6.360m x 9.353).
- 120mm thick, 52MPa concrete;
- Edges vertically supported.
- A142 mesh (142 mm^2 per metre, 580MPa steel at 200mm spacing in x and y directions) at 69mm effective depth;
- Mesh ductility 12%.

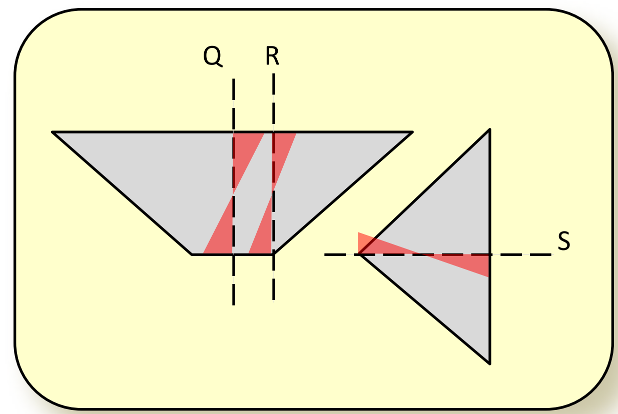
Garston comparison for different aspect ratios



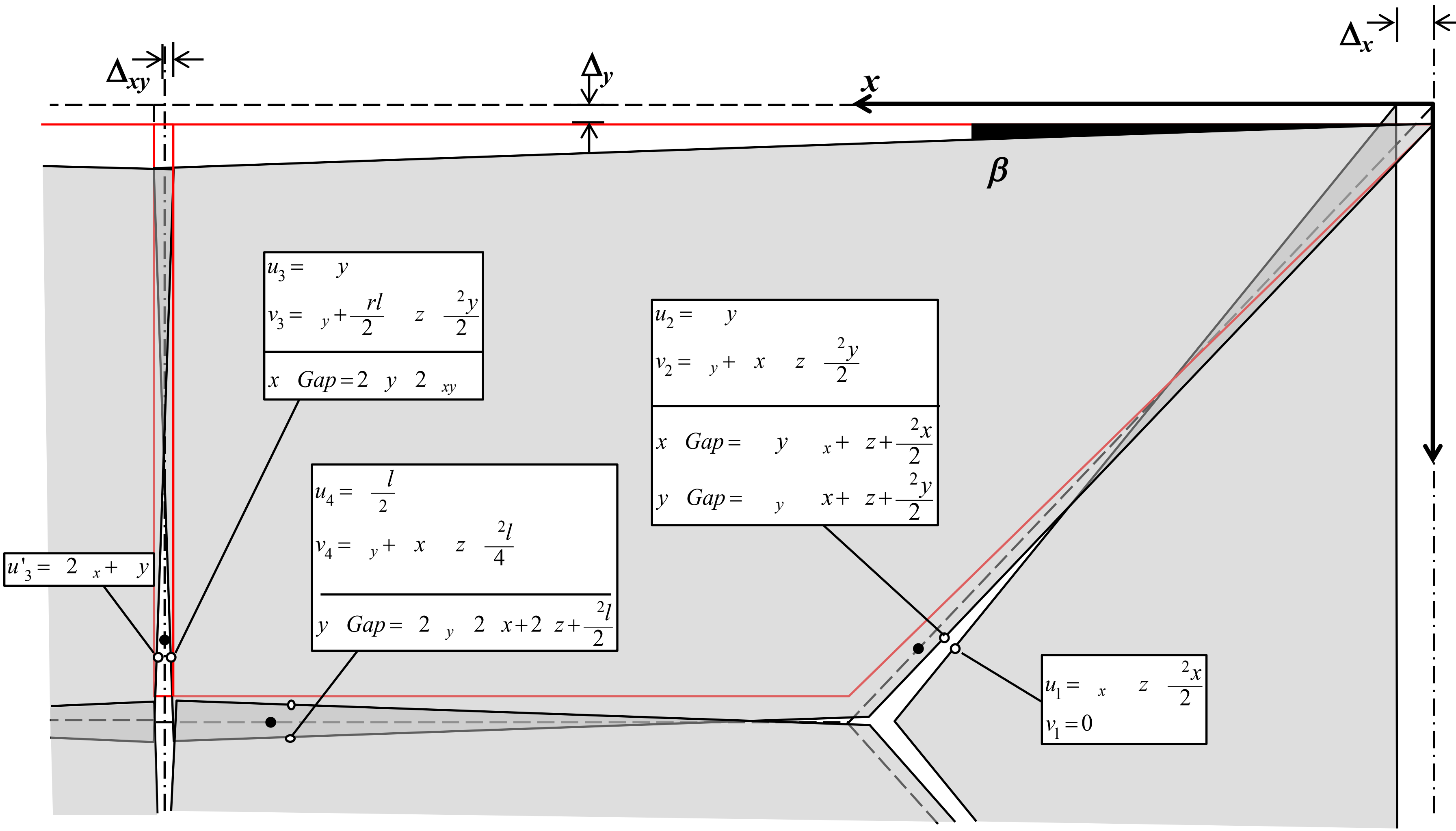
Garston – apply tensile strength to change mechanism



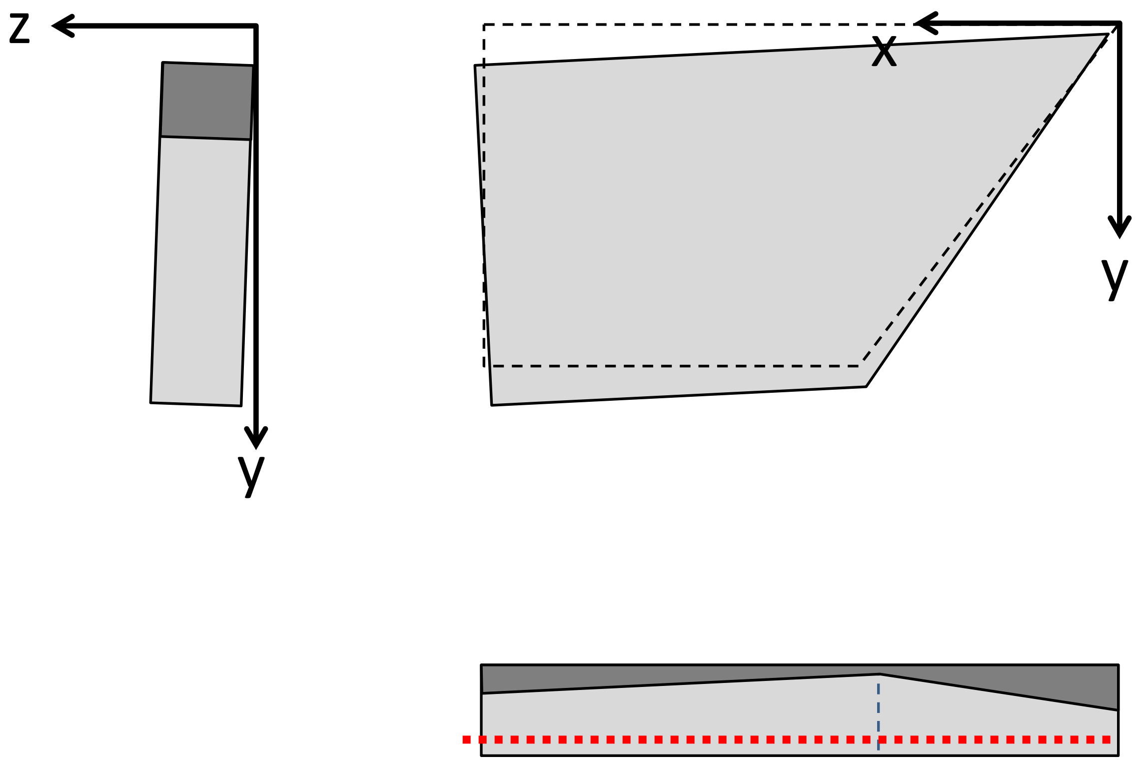
EC2 tensile strength for C52 [$0.3f_c^{0.67}$]



New mechanism – central through-depth crack



Change of stress blocks – ductile y-reinforcement



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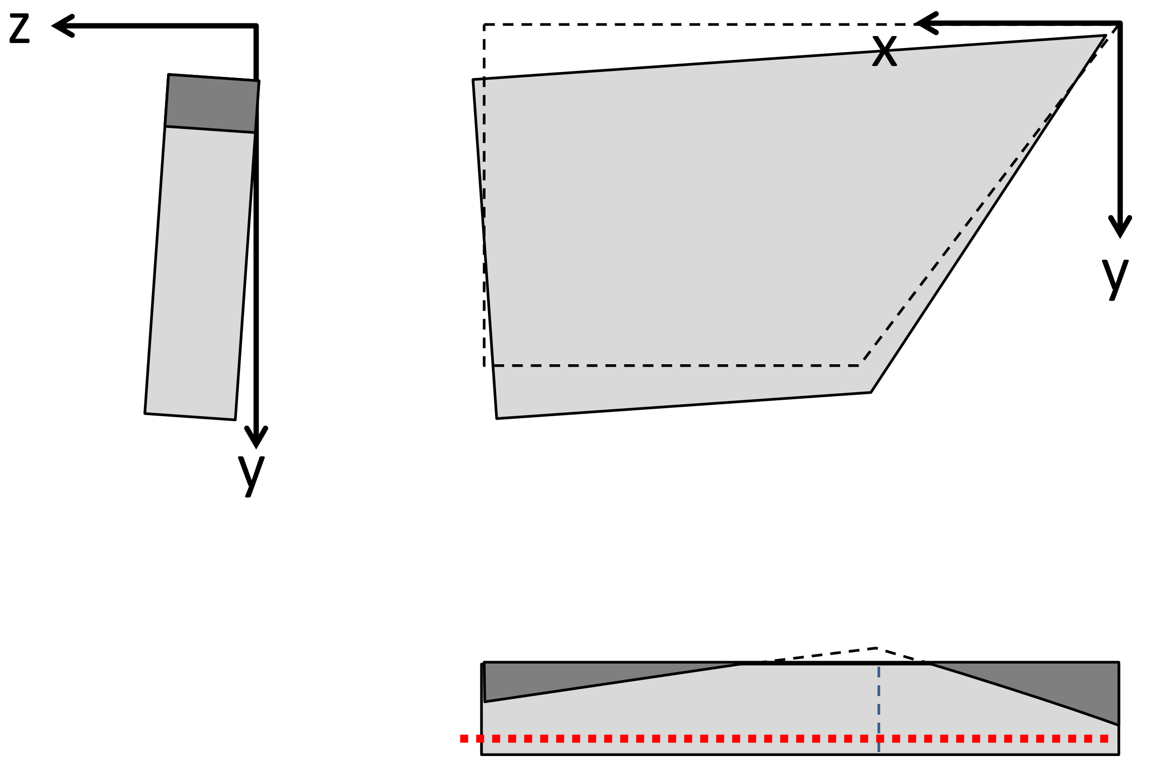

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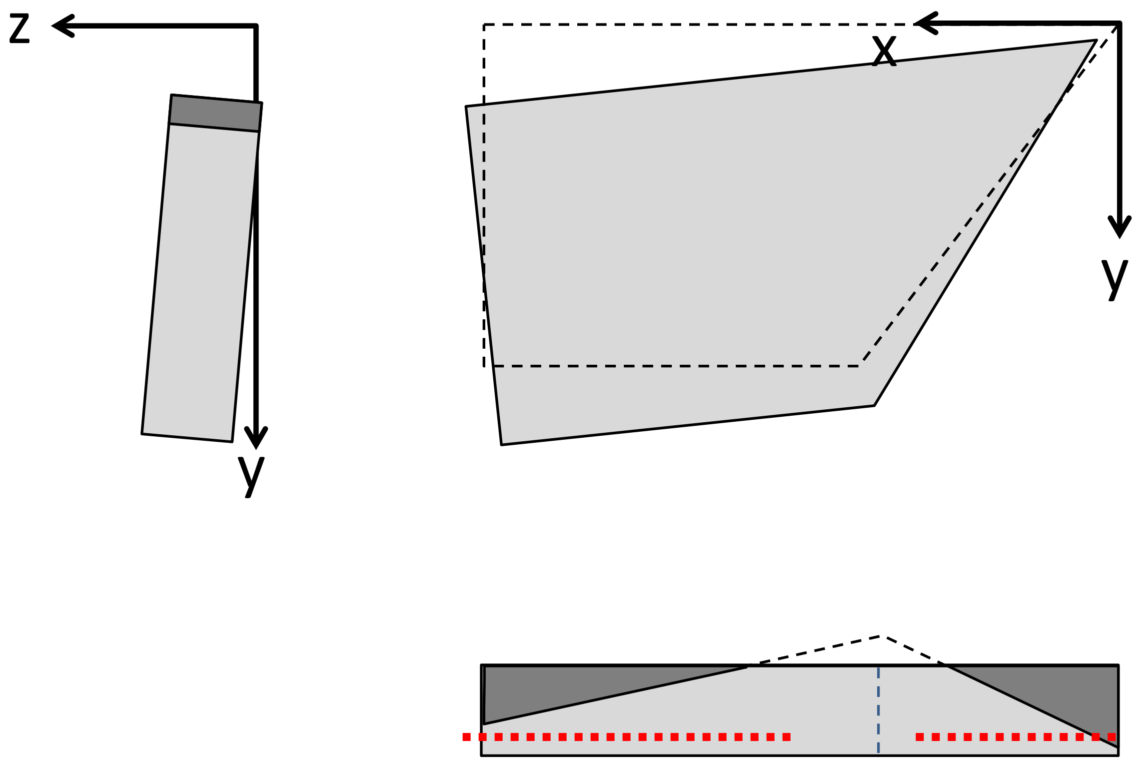
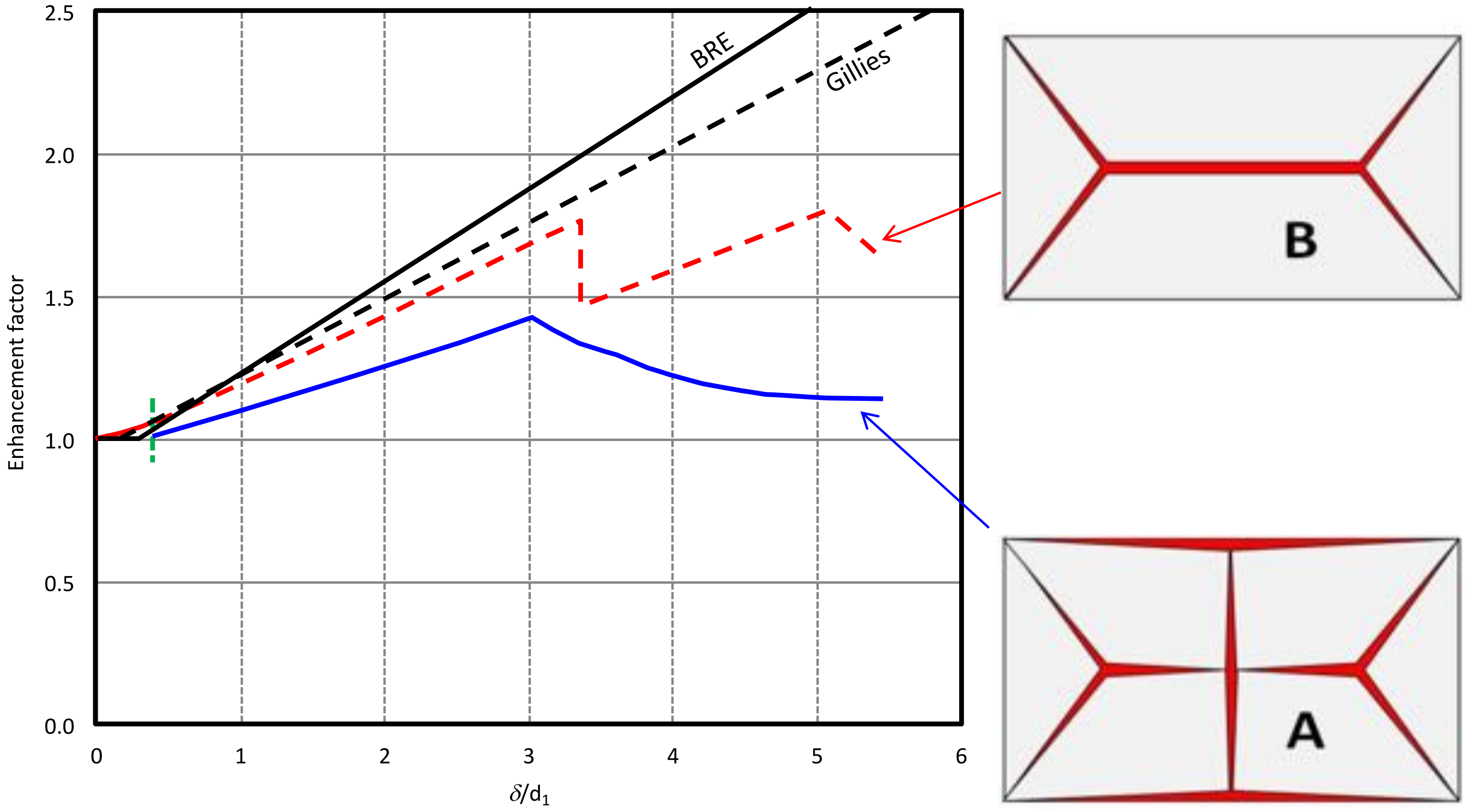


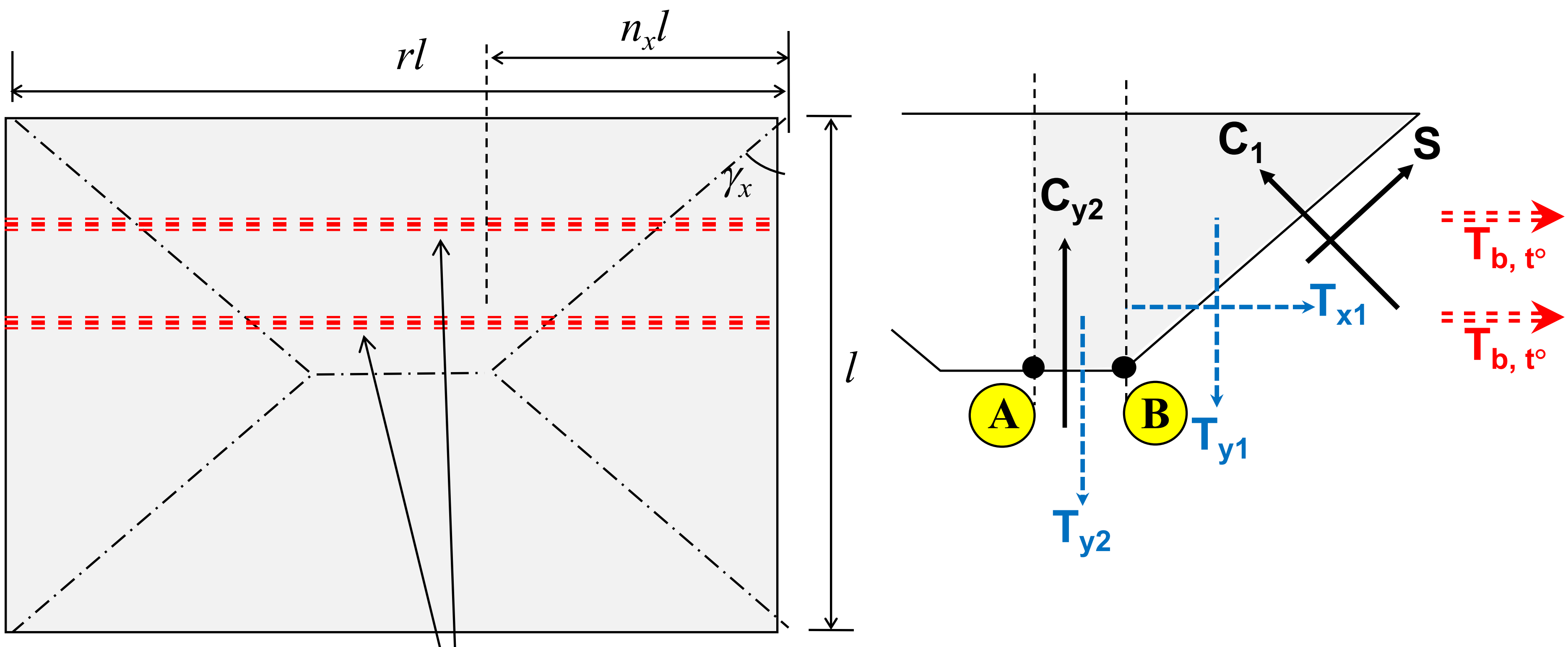
From basic mechanism to centrally cracked



With attached steel beams …

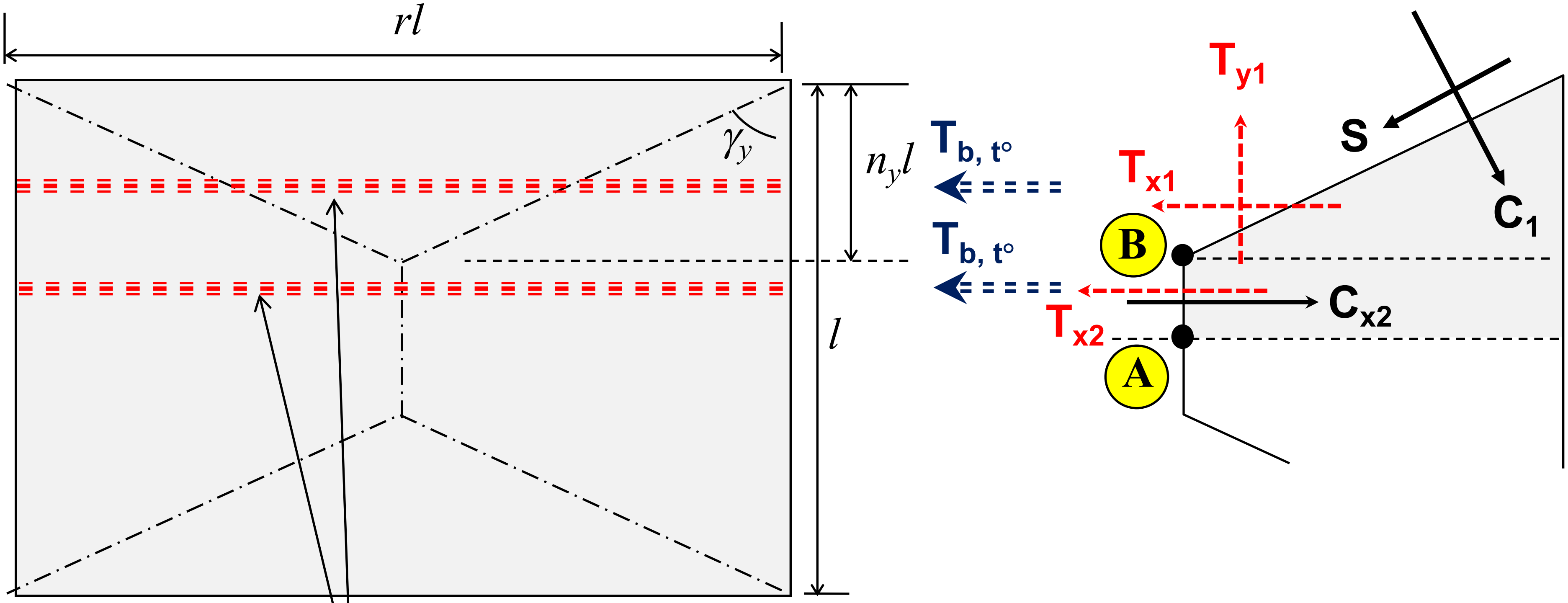
… the yield-line mechanism changes.

Forces on the x-aligned mechanism



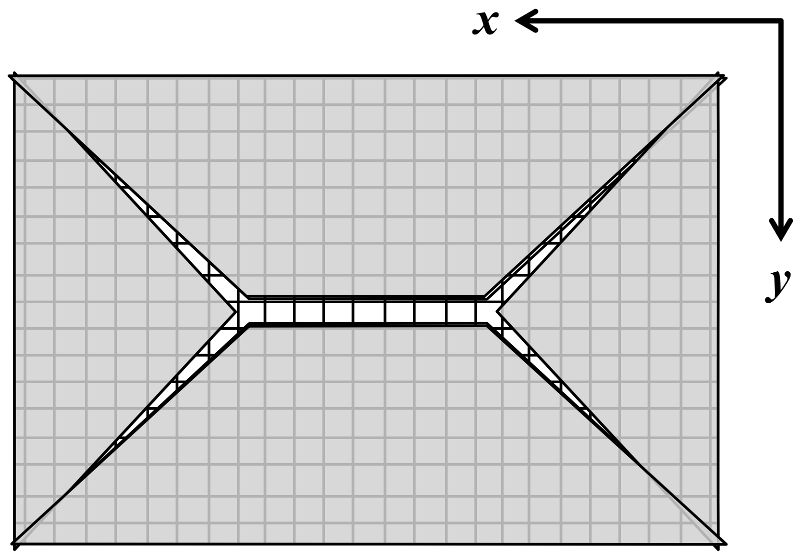
Unprotected beams at high temperature

Forces on the y-aligned mechanism



Unprotected beams at high temperature

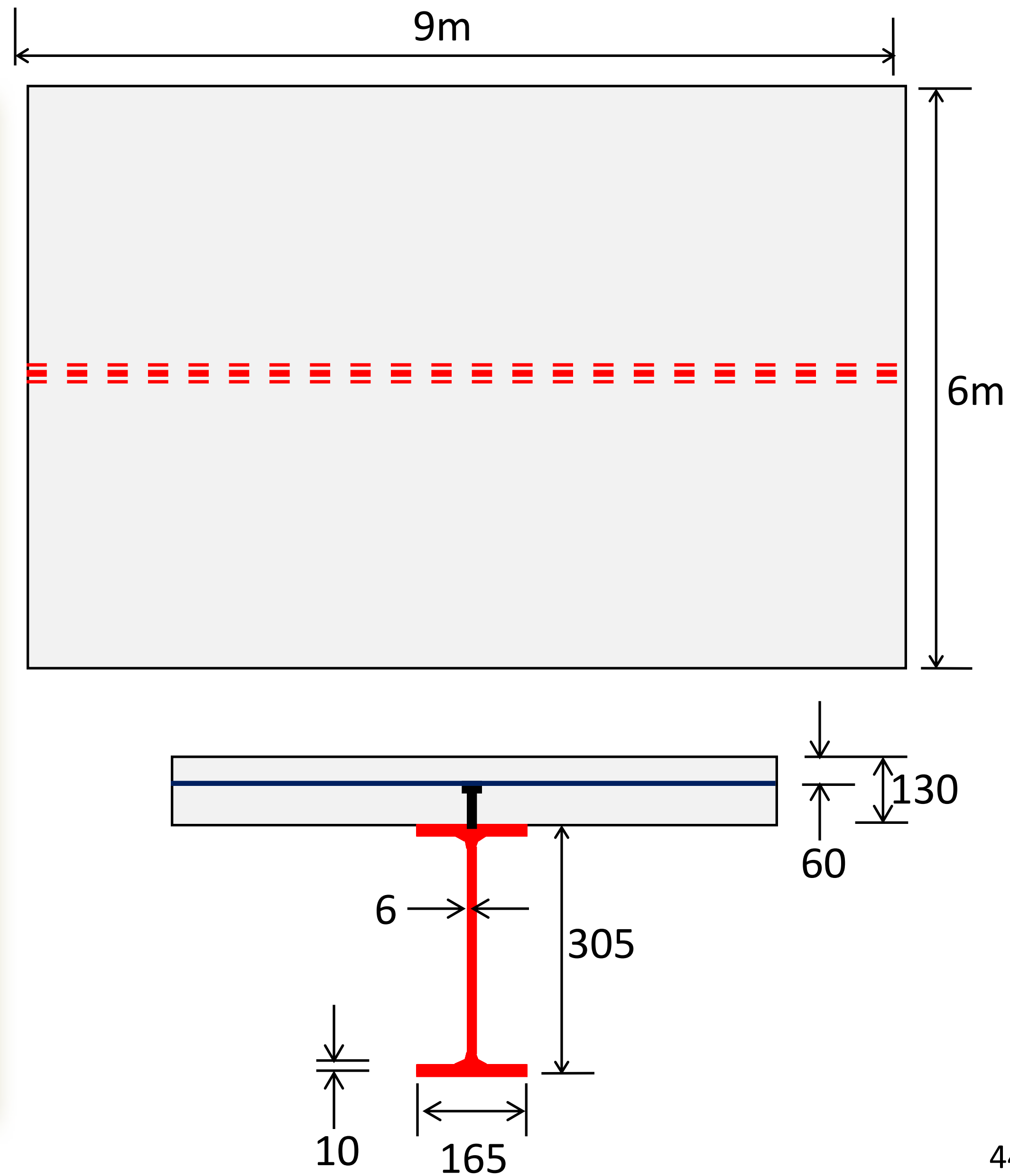
Combinations of compression block and rebar fracture



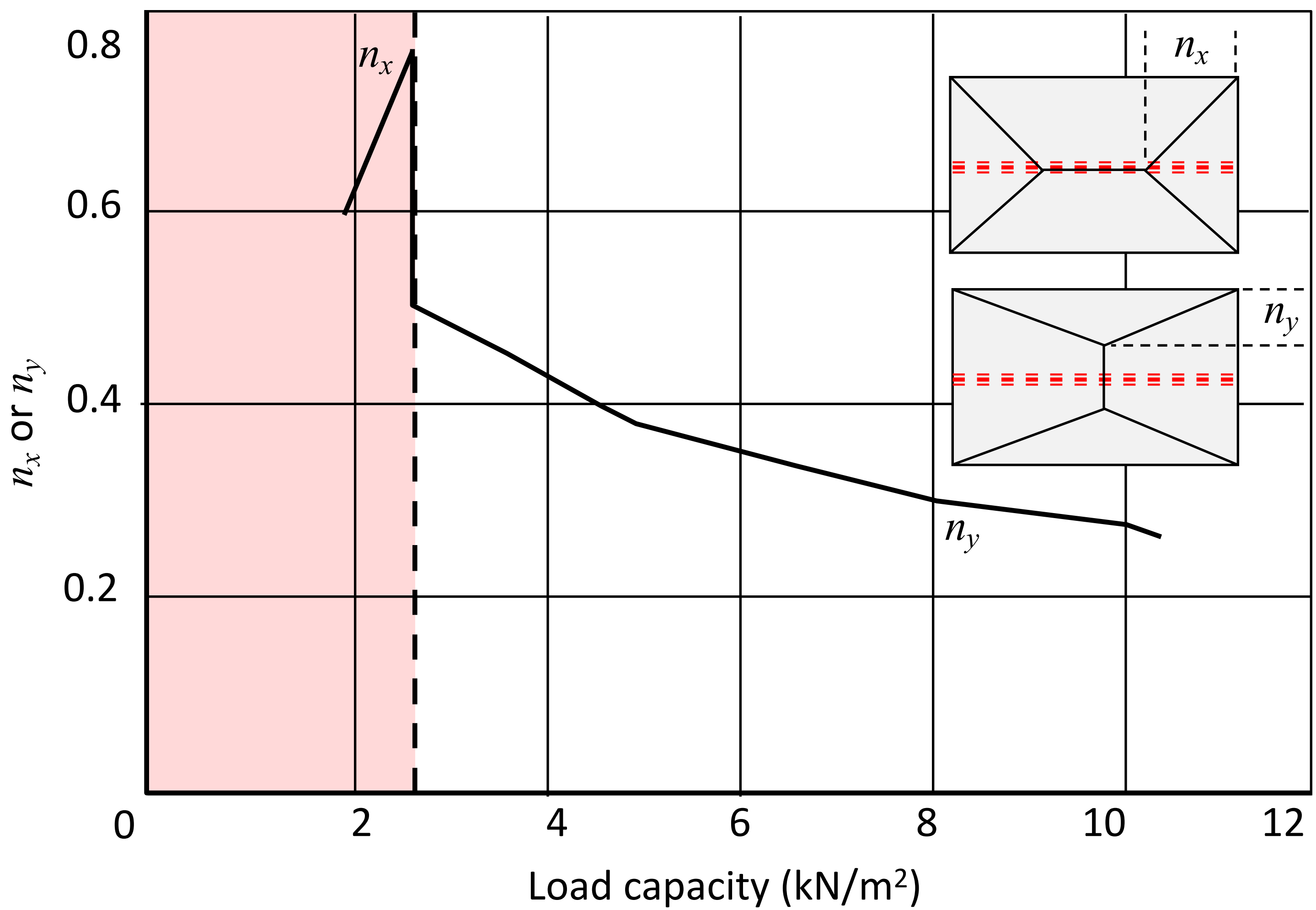
Compression block		Reinforcement mesh fracture level (*x*-aligned mechanism)					
		None	Central y	Diag. x	Central + Diag. y	Central + Diag. x	Central + Diag. x, y
Full	**above mesh**	a1	a1’	a1*	a1**	a1*’	a1***
	below mesh	a2	a2’	a2*	a2**	a2*’	a2***
Triangular	**above mesh**	b1	b1’	b1*	b1**	b1*’	b1***
	below mesh	b2	b2’	b2*	b2**	b2*’	b2***
Trapezoidal		c1	c1’	c1*	c1**	c1*’	c1***

Example of application: 9m x 6m composite slab

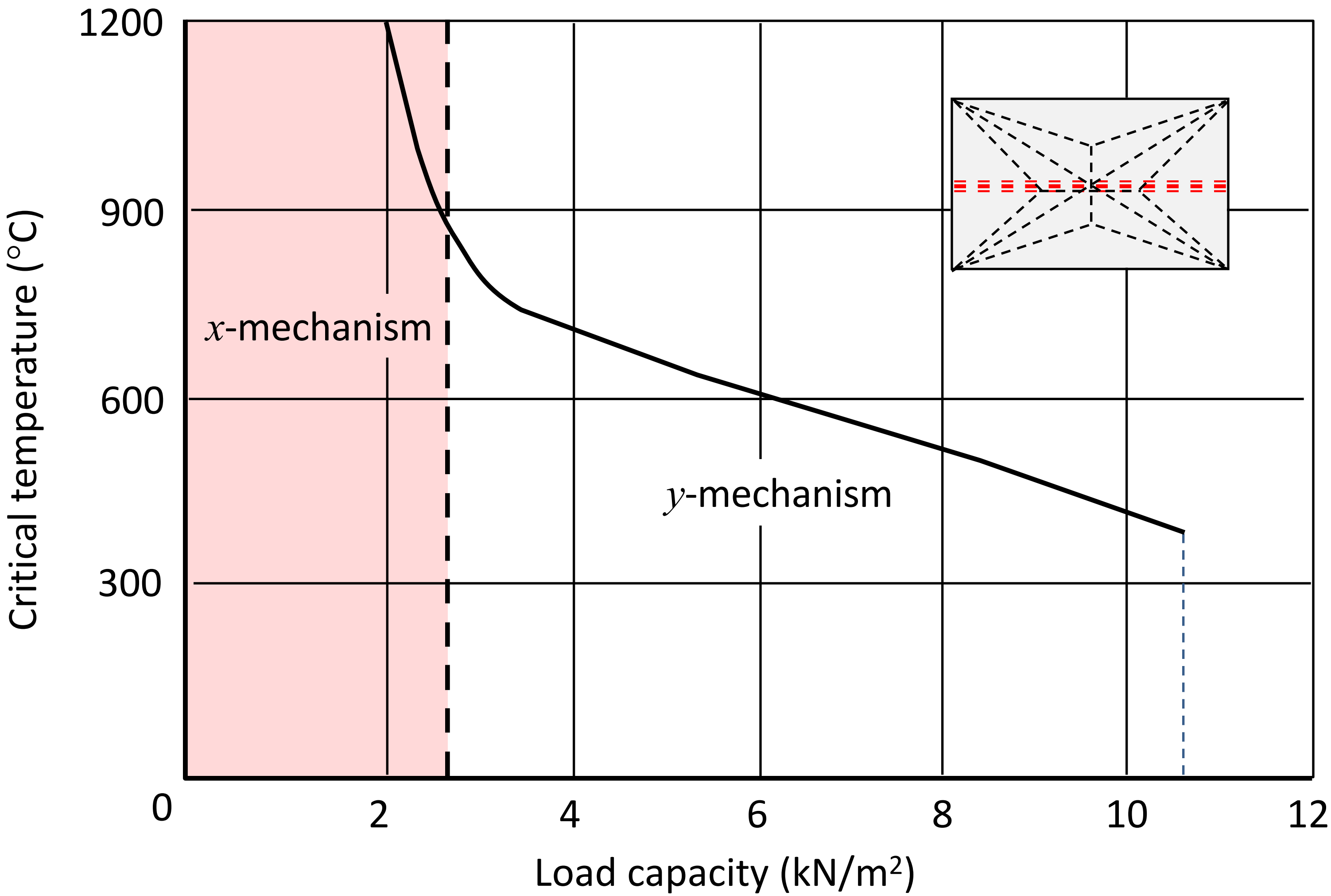
- 130mm thick slab, 30MPa concrete;
- A142 mesh (142 mm^2 per metre, 500MPa steel at 200mm spacing in x and y directions) at 60mm effective depth;
- Mesh effective ductility (over 200mm length) 1%: fracture crack-width 2mm;
- One central downstand steel beam, 305x165UKB40, Grade S275 - unprotected against fire;
- Edges vertically supported.



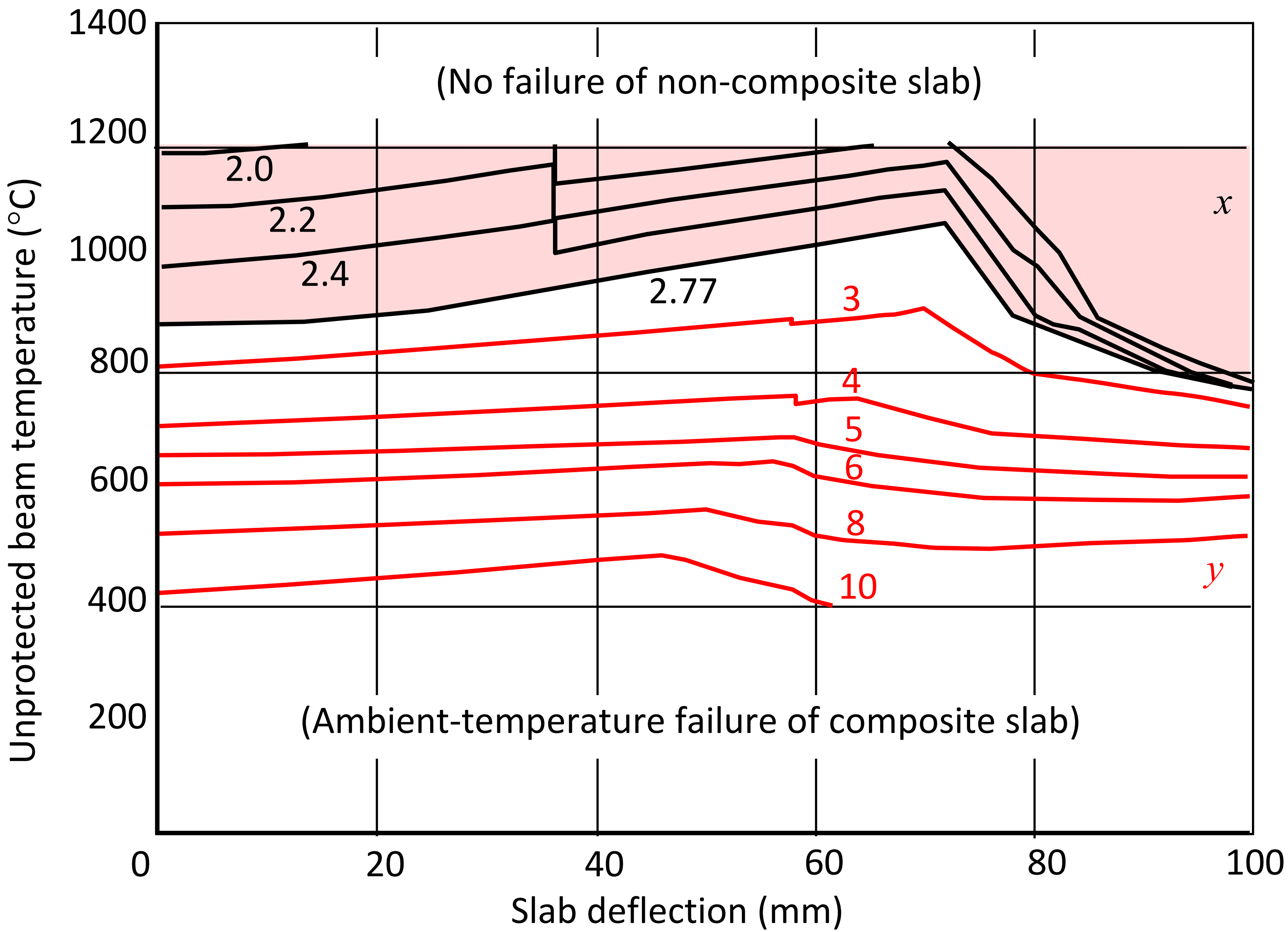
Initial yield-line parameter for different load capacities



Critical temperature variation with load capacity



Enhancement of critical steel temperature with deflection



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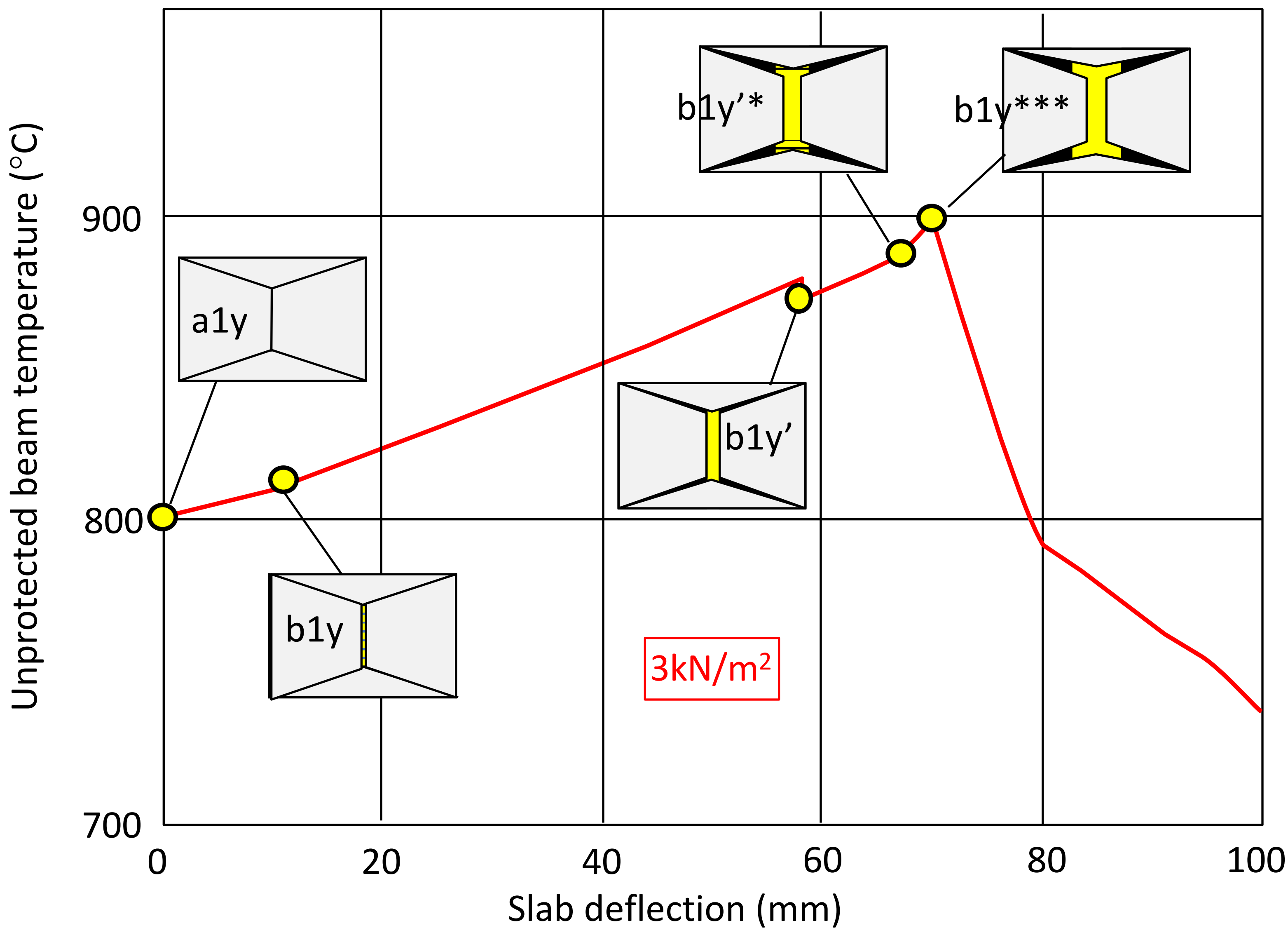
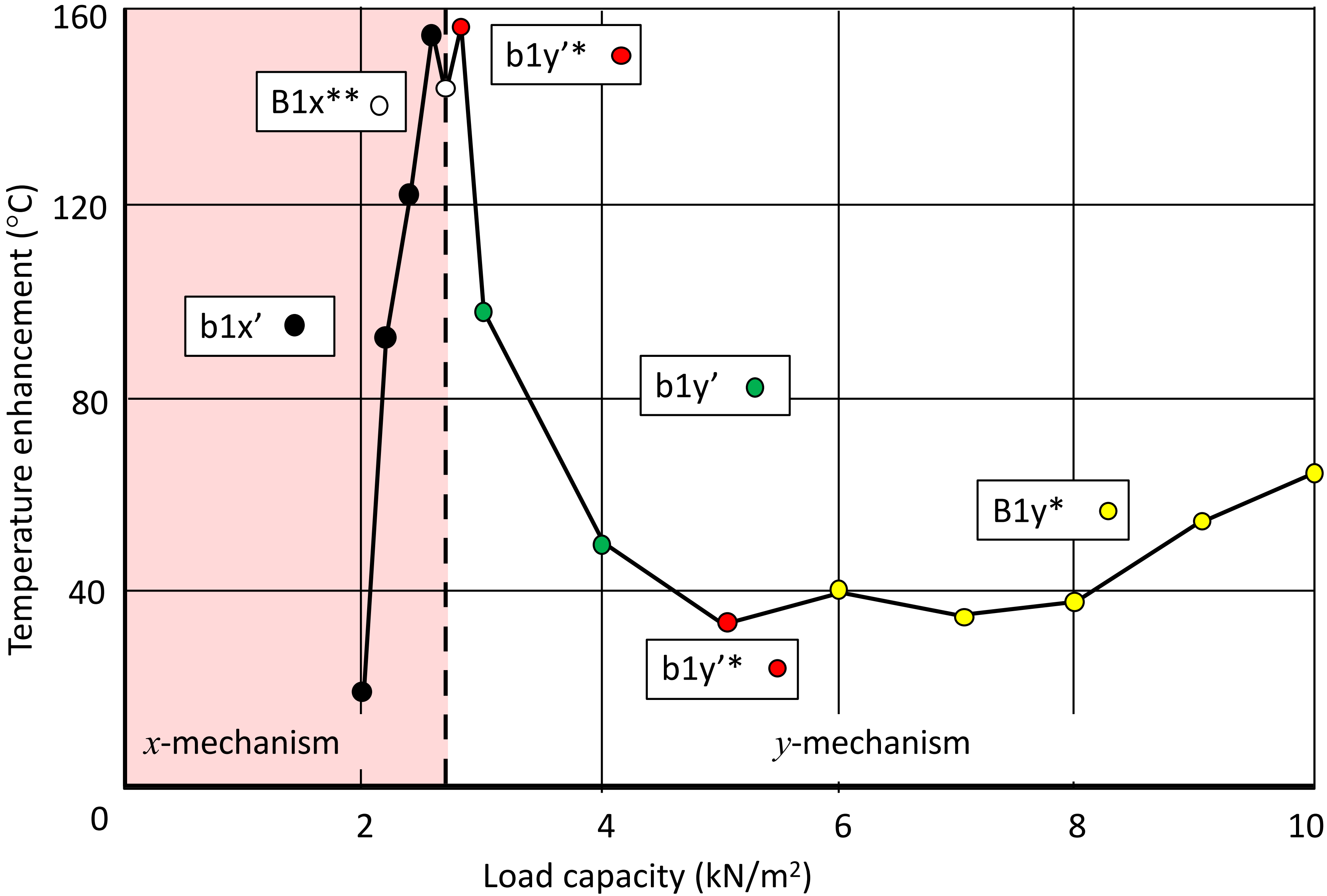
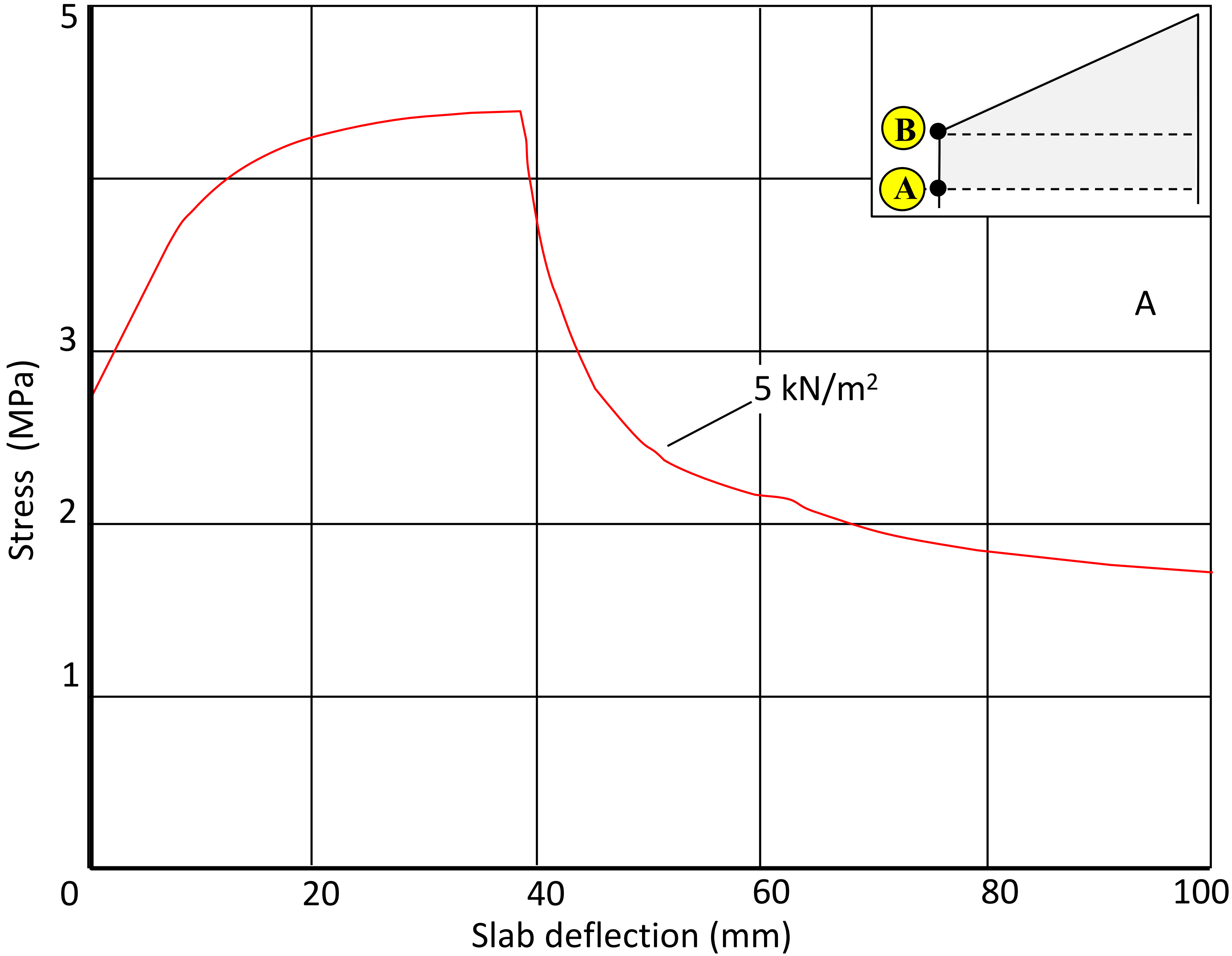


Maximum steel temperature enhancements



Maximum tensile stress at section A



Existing simplified methods:

For concrete slabs:

- Fixed membrane traction distribution – independent of slab deflection.
- Membrane traction distribution only valid while concrete has compression along whole yield lines.
- Assumes central crack fully formed. Rebar at ultimate strength (+10%)
- Enhancement factor starts below 1.0.

For composite slabs in fire:

- Yield-line pattern based on non-composite slab.
- Superposes high-temperature composite beam capacity and deflection-controlled slab enhancement.
- Criterion for mid-span through-depth crack is meaningless.

In summary …

The new approach:

For all slabs:

- Based on the kinematics of deflecting flat facets of the small-deflection yield line mechanism, together with in-plane equilibrium of the concrete and steel forces.
- Allows concrete stress blocks to move and mesh to fracture across yield lines.

For composite slabs in fire:

- Keeps load constant, allows beams temperature to increase until yield line mechanism forms.
- Enhancement of steel beam temperature with deflection.

Biggest problems to be solved:

- Fracture ductility of rebar across discrete cracks - yield lines or through-depth mid-span crack.
- Concrete tensile stress to initiate the mid-span (or intersection) crack.

Thank you